Modeling and solving job shop problems with complex process features and complicated objectives

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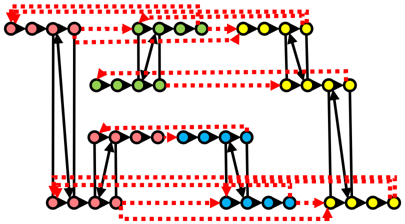

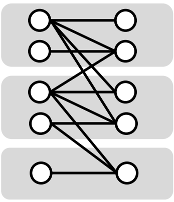

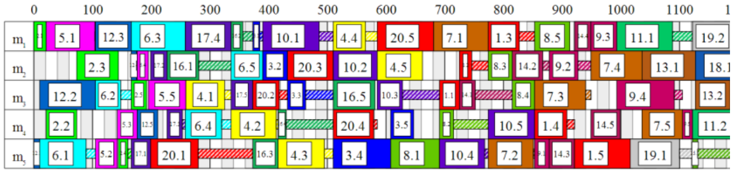

Seminar @ GERAD

Oct 27, 2016

Overview

1. The Classical Job Shop Scheduling Problem

1. Introduction
2. A Combinatorial Formulation in a Disjunctive Graph
3. Applications in Practice

2. Complex Process Features

1. Some Process Features
2. An Application: The BJS-RT and ALPHABOT

3. Complicated Objectives

1. General Regular Objective
2. A Class of Convex Cost Objectives

4. A Local Search Solution Approach

1. The Job Insertion Problem
2. Local Moves and Locally Improving Moves
3. Some Computational Results

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1. The Classical Job Shop Scheduling Problem

1.1. Introduction

- The classical job shop scheduling problem
  - A fundamental optimization problem in operations research
  - NP-hard and one of the most computationally stubborn combinatorial problem (Applegate and Cook, 1991)
  - Addressed by numerous researchers, and a large body of knowledge accumulated over the last 60 years

- Given
  - A set M of machines (sometimes called resources)

    $M=\{m_1,m_2,m_3\}$
  - A set I of operations (sometimes called activities)

    $I=\{1,2,3,4,5,6,7,8,9,10\}$
  - A set $\mathcal{J}$ of jobs and

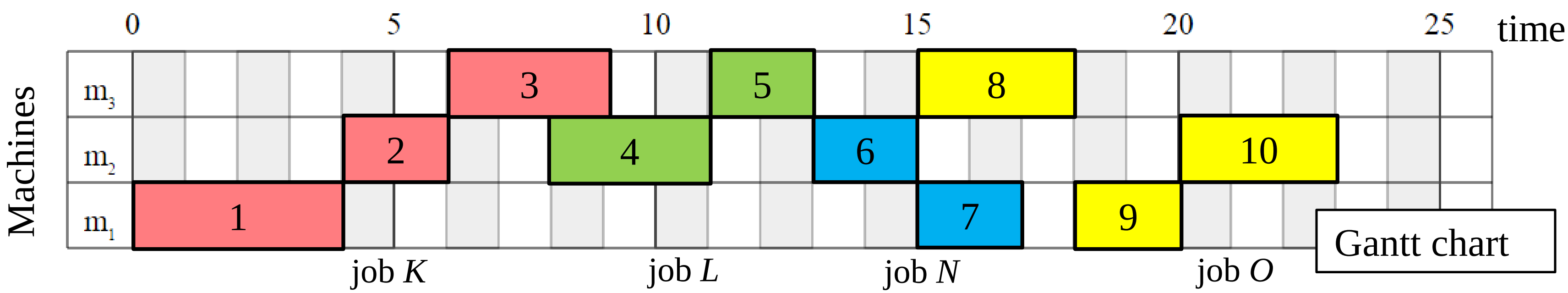
    $\mathcal{J}=\{K,L,N,O\}$
  - A job $J \in \mathcal{J}$ is an ordered set of operations $J = (J_1,J_2,\ldots J_{|J|})$, $J_i \in I$ for $i \in \{1,\ldots|J|\}$, specifying a processing sequence

    $K=(1,2,3)$ $L=(4,5)$ $N=(6,7)$ $O=(8,9,10)$
    - (Note: $\mathcal{J}$ is a partition of I, i.e. each operation is in exactly one job)
  - Each operation $i \in I$ executed on one machine m_i during p_i time units

- Data of the example:

Proc. time p	*First Op.*	*Sec. Op.*	*Third Op.*
Job K	m_1, 4	m_2, 2	m_3, 3
Job L	m_2, 3	m_3, 2	-
Job N	m_2, 2	m_1, 2	-
Job O	m_3, 3	m_1, 2	m_2, 3

- Find: A feasible schedule, i.e. a starting time of each operation:
  - Respect the processing sequences within the jobs
  - Each machine is used at most by one operation at any time ("unit capacity")
  - No preemption (no interruption) of the processing of an operation
- A visual representation of a schedule



- Objective: minimize the makespan (i.e., total duration)

A Problem Formulation

- Many different mathematical formulations exist
  - A main characteristic: continuous-time / discrete-time (time-indexed)
- Some references
  - Ku, Wen-Yang, and J. Christopher Beck. Mixed integer programming models for job shop scheduling: A computational analysis. *Computers & Operations Research* 73 (2016): 165-173
  - Brucker, P., & Knust, S. (2011). *Complex Scheduling*. Springer

"Introduction to Constraint Programming"
30 Oct - 1 Nov, Université Concordia

- A continuous-time formulation
  - Based on: Manne, A. (1960). On the job-shop scheduling problem. *Operations Research*, 8(2), 219–223
  - Introduce fictive end operation τ
  - Call two operation i and j *consecutive* if j follows i in some job
  - For each machine $m \in M$, let I_m be the set of operations executed on m
  - For each operation $i \in I \cup \{\tau\}$, α_i denotes the (variable) starting time

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Disjunctive programming formulation

$$
\begin{aligned}
&\text{minimize} \quad \alpha_\tau \\
&\text{subject to:} \\
&\alpha_j - \alpha_i \ge p_i \quad \text{for all } i \text{ and } j \text{ consecutive in a job} \\
&\alpha_\tau - \alpha_i \ge p_i \quad \text{for all } i \in I \\
&\alpha_j - \alpha_i \ge p_i \ \text{ OR } \ \alpha_i - \alpha_j \ge p_j \quad \text{for all } i,j \in I_m \\
&\alpha_i \ge 0 \quad \text{for all } i \in I \cup \{\tau\}
\end{aligned}
$$

min. end time

job structure

end operation

machine capacity

start after 0

replace OR operator by:

Mixed-integer linear progr. formulation

$$
\begin{aligned}
&\text{minimize} \quad \alpha_\tau \\
&\text{subject to:} \\
&\alpha_j - \alpha_i \ge p_i \quad \text{for all } i \text{ and } j \text{ consecutive in a job} \\
&\alpha_\tau - \alpha_i \ge p_i \quad \text{for all } i \in I \\
&\boxed{\begin{aligned}&\alpha_j - \alpha_i \ge p_i - M(1 - z_{ij}), \\ &\alpha_i - \alpha_j \ge p_j - M z_{ij} \quad \text{for all } i,j \in I_m\end{aligned}} \\
&\alpha_i \ge 0 \quad \text{for all } i \in i \in I \cup \{\tau\} \\
&\boxed{z_{ij} \in \{0,1\} \quad \text{for all } i,j \in I_m}
\end{aligned}
$$

z_{ij}: 1 if i is before j, and 0 otherwise
M: large constant, here e.g., $M = \sum_{i \in I} p_i$

1.2. A Combinatorial Formulation in a Disjunctive Graph

- Constraints have all the same structure: $\alpha_w - \alpha_v \geq d_{vw}$ Difference of two times

  Precedence Constraints

- Simplify the formulation by introducing a disjunctive graph

Disjunctive programming formulation

minimize $\alpha_\tau - \alpha_\sigma$

subject to: a constant

$\alpha_j - \alpha_i \geq p_i$ for all i and j consecutive in a job — conjunctive

$\alpha_\tau - \alpha_i \geq p_i$ for all $i \in I$

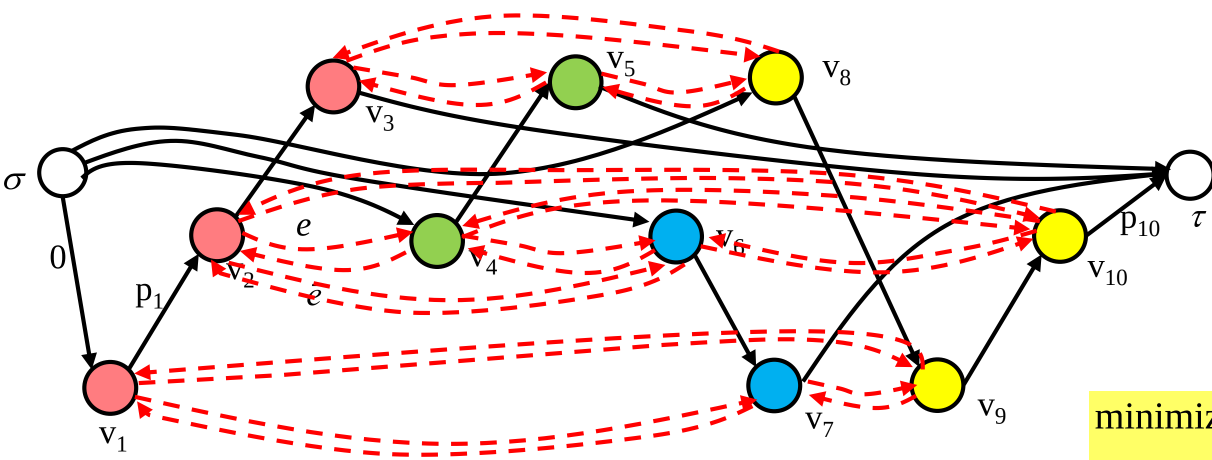
$\alpha_j - \alpha_i \geq p_i$ OR $\alpha_i - \alpha_j \geq p_j$ for all $i,j \in I_m$ — disjunctive

$\alpha_i \geq 0$ for all $i \in I \cup \{\tau\}$ — conjunctive

$-\alpha_\sigma$

(and $\alpha_\sigma = 0$, which is, actually, simple to fulfill)

$G=(V,A,E,\mathcal{E},d)$



(some redundant arcs incident to σ and τ omitted)

- Each operation is repres.by a node
- Each precedence constraint $x_w - x_v \geq d_{vw}$ represented by an arc (v,w) with weight d_{vw}
- Set of conjunctive arc A and
- Set of disjunctive arcs E
- Disjunctive structure $\mathcal{E}$: consists of pairs of disjunctive arcs $\{e, \bar{e}\} \in \mathcal{E}$

minimize $\alpha_\tau - \alpha_\sigma$

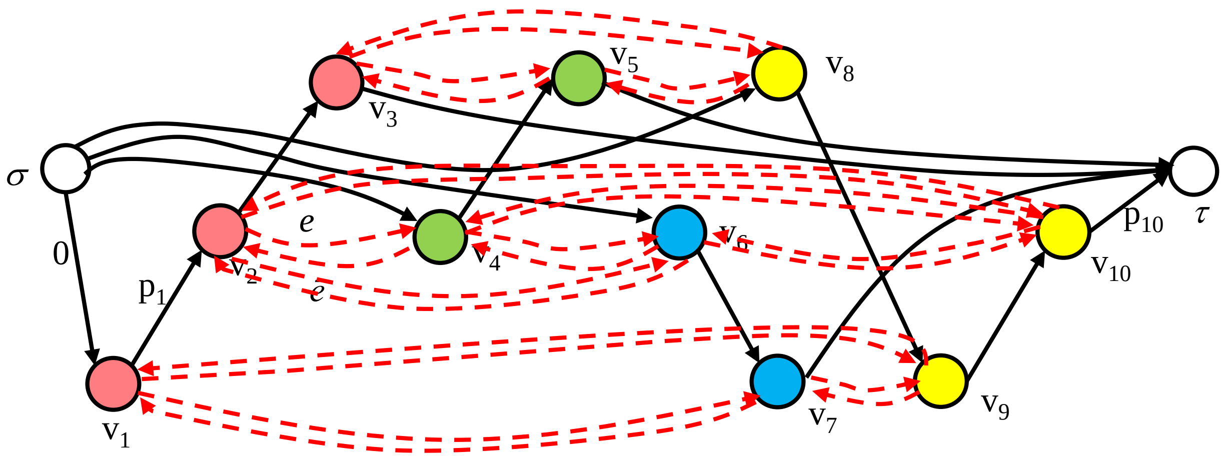
subject to:

$\alpha_w - \alpha_v \geq d_{vw}$ for all $(v,w) \in A$

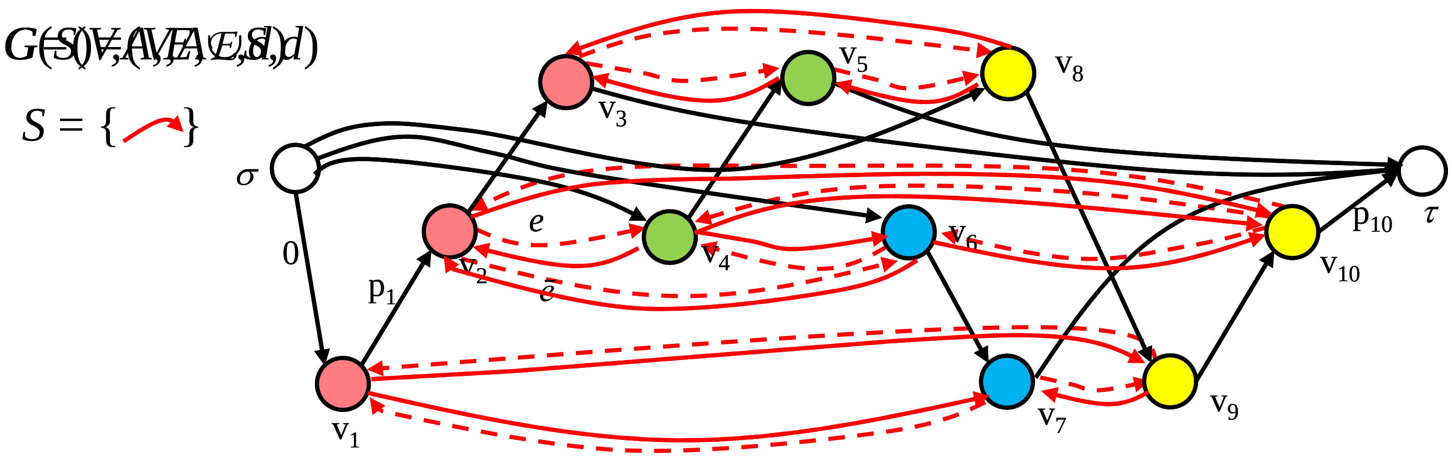
$\alpha_w - \alpha_v \geq d_{vw}$ OR $\alpha_{v'} - \alpha_{w'} \geq d_{v'w'}$

for all $\{(v,w),(v',w')\} \in \mathcal{E}$

$G=(V,A,E,\mathcal{E},d)$



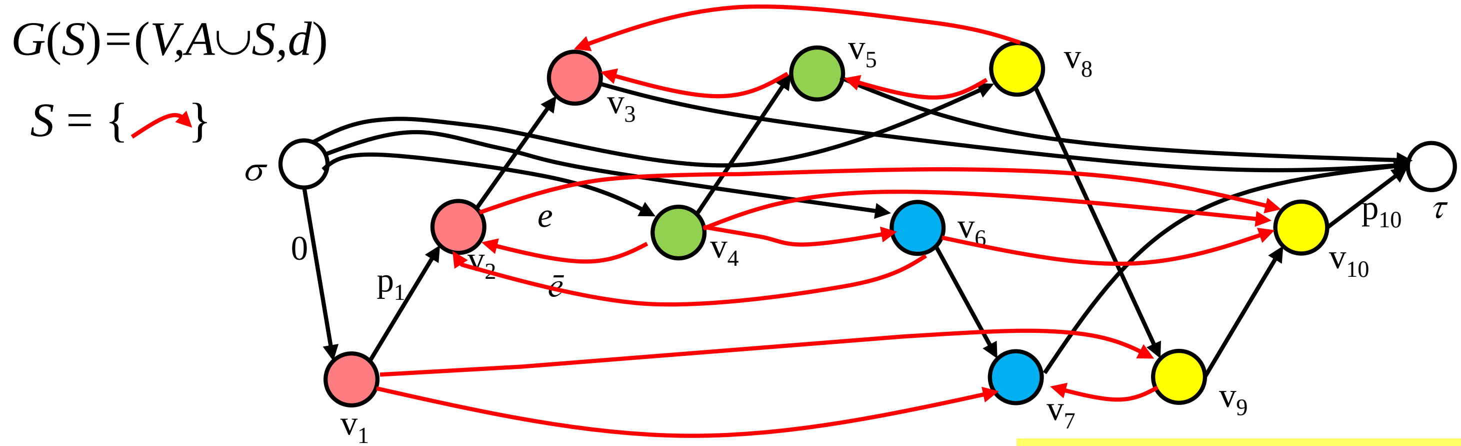
Selections



- Capturing solutions:

Definition
A selection: any set $S \subseteq E$ of disjunctive arcs
A selection S is complete if $S \cap \{e,\bar{e}\} \neq \emptyset$ for all $\{e,\bar{e}\} \in \mathcal{E}$
A selection S is positive acyclic if the graph G(S)=(V,$A\cup S$,d) has no cycle of positive length and positive cyclic otherwise.

Timing Problem: Determine Starting Times



- For any selection S, we have the following timing problem at hand:
  - A potential problem!

$$\text{minimize} \quad \alpha_\tau - \alpha_\sigma$$
$$\text{subject to:}$$
$$\alpha_w - \alpha_v \geq d_{vw} \quad \text{for all } (v,w) \in A \cup S$$

- This is a (special) LP with the following dual:

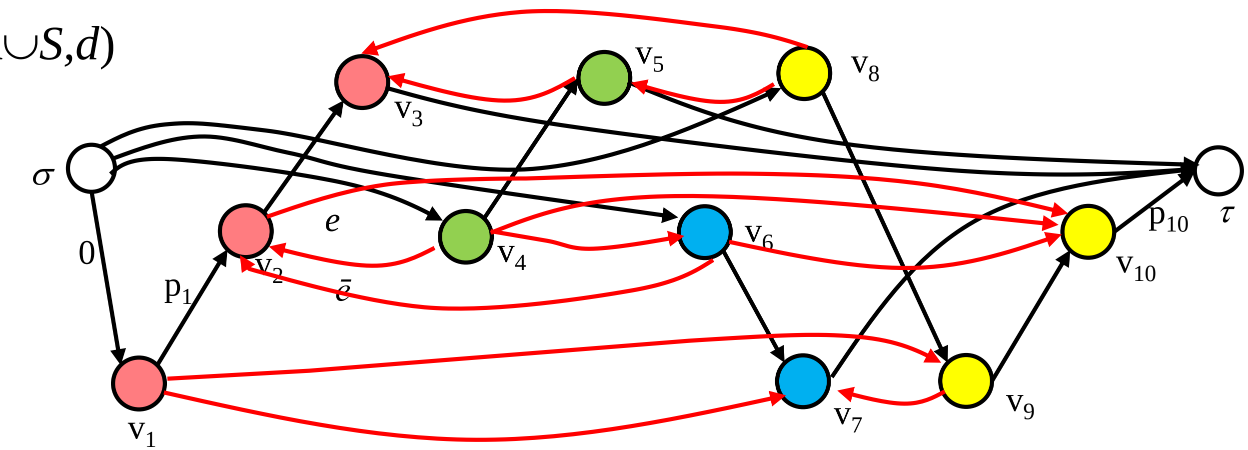
$$\text{maximize} \quad \sum_{(v,w)\in A\cup S} d_{vw} x_{vw}$$
$$\text{subject to:}$$
$$\sum_{w:(w,v)\in A\cup S} x_{wv} - \sum_{w:(v,w)\in A\cup S} x_{vw} = \begin{cases} 1 & \text{for } v = \tau \\ -1 & \text{for } v = \sigma \\ 0 & \text{for } v \in V - \{\sigma,\tau\} \end{cases}$$
$$x_{vw} \geq 0 \text{ for all } (v,w) \in A \cup S$$

A (simple) network flow problem!

- Well-known and easy to see: feasible times exist if and only if S is positive acyclic
- Hence, selection S called feasible if S is complete and positive acyclic

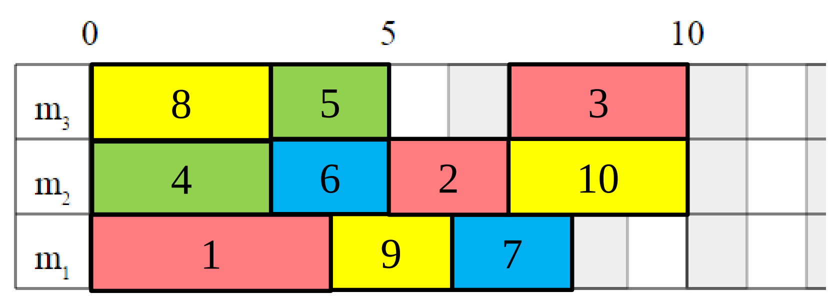
$G(S)=(V,A \cup S,d)$

$S = \{\ \}$



- For any feasible selection S, an optimal solution can be found by
  - Computing the earliest time schedule $\alpha(S)$ where
  - For all $v \in V$, $\alpha_v(S)$: length of a longest path from σ to node v in $G(S)$
  - Can be done by topological sorting algorithm, time complexity: $O(n+m)$
- Timing problem efficiently solvable!

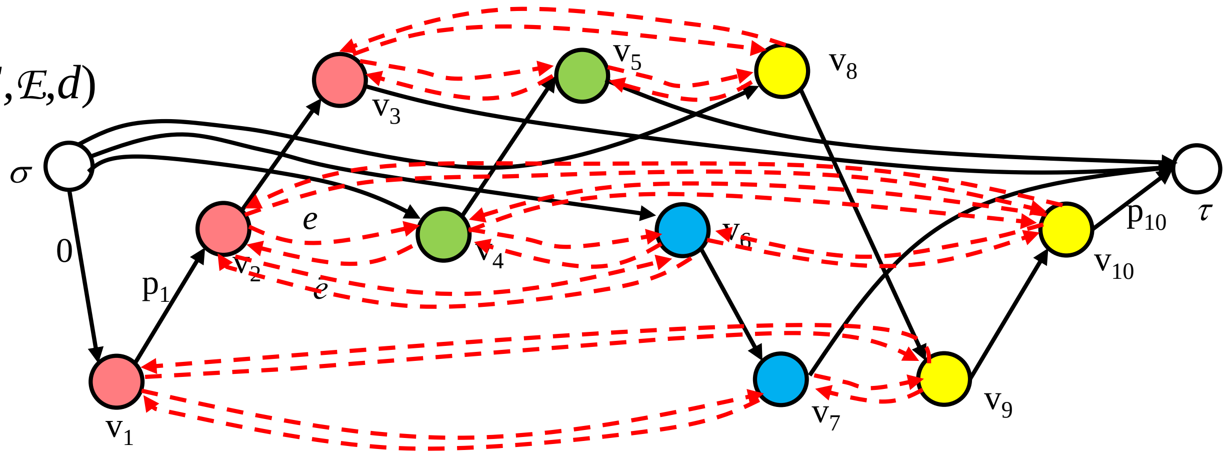
Earliest time schedule of the selection above:



Makespan: 10

A Combinatorial Problem Formulation

$G=(V,A,E,\mathcal{E},d)$



Among all feasible selections, find a selection S minimizing the length of a longest path from σ to τ in $G(S)$.

1.3. Applications in Practice





Liebherr: construction machines, factory in Telfs, Austria

Source: https://www.youtube.com/watch?v=V-37jTfLe8o

Flexible Manufacturing Systems and Robotic Cells

Flexible manufacturing system (Kuka)

Source: http://www.kuka-systems.com/NR/exeres/73B56636-DC3A-4CD8-A7E7-9D9E15345AB1

A robotic cell:

Source: http://www.canadianmetalworking.com/features/where-are-your-robots/

Logendran, R., & Sonthinen, A. (1997). A tabu search-based approach for scheduling job-shop type flexible manufacturing systems. *Journal of the Operational Research Society*, *48*(3), 264–277.

Hall, N. G., Kamoun, H., & Sriskandarajah, C. (1997). Scheduling in robotic cells: classification, two and three machine cells. *Operations Research*, *45*(3), 421–439.

Hoist Scheduling and Factory Cranes

Electroplating plant for surface treatment
Source: Surface Technology Solutions (stsindustrie.com)

A paper-roll shipping store with automated transports
Source: liftandhoist.com

Leung, J. M. Y., Zhang, G., Yang, X., Mak, R., & Lam, K. (2004). Optimal cyclic multi-hoist scheduling: a mixed integer programming approach. *Operations Research*, *52*(6), 965–976.

Peterson, B., Harjunkoski, I., Hoda, S., & Hooker, J. N. (2014). Scheduling multiple factory cranes on a common track. *Computers and Operations Research*, *48*, 102–112.

Gantry Crane Scheduling

A rail-road terminal with gantry cranes
Source: dradio.de

Automated stacking cranes in a storage area
Source: wcms.demagrobots.info

Li, W., Wu, Y., Petering, M. E. H., Goh, M., & de Souza, R. (2009). Discrete time model and algorithms for container yard crane scheduling. *European Journal of Operational Research*, *198*(1), 165–172.

Ng, W. C. (2005). Crane scheduling in container yards with inter-crane interference. *European Journal of Operational Research*, *164*(1), 64–78.

Train Scheduling

Trains
Source: www.thechronicle.com.au

Metro Stations
Source: en.wikipedia.org/wiki/Duomo_(Milan_Metro)

Liu, S., & Kozan, E. (2011). Scheduling trains with priorities: a no-wait blocking parallel-machine job-shop scheduling model. *Transportation Science*, *45*(2), 175–198

Mannino, C., & Mascis, A. (2009). Optimal Real-Time Traffic Control in Metro Stations. *Operations Research*, *57*(4), 1026–1039

Gap Between Theory and Practice

- However, the classical job shop rarely applicable in practice
- Process features that are not captured
  - Sequence-dependent setup times
    - Cleaning
    - Idle moving of robots
  - Routing flexibility
    - Multiple machines of the same type
  - Storage time restrictions
    - Chemical treatments
  - Storage space restrictions
    - Train is always on a rail section (= machines)
    - Robotic cells (predefined or no storage space)

- More complicated objectives than the makespan
  - Work-in process related
    - Total (weighted) flow time
  - Tardiness related
    - Total (weighted) linear tardiness costs
    - Total (weighted) squared tardiness costs
    - Number of tardy jobs
  - Earliness and tardiness related
    - "just-in-time objectives": sum of earliness and tardiness costs, possibly non-linear
- Numerous new job shop scheduling models appeared
  - Mostly, capturing just few features
  - Specific, application-oriented solution approaches
  - Job shop scheduling research has become fragmented, and scheduling software highly specialized (Bulbul, K., & Kaminsky, P. (2013). A linear programming-based method for job shop scheduling. Journal of Scheduling, 16(2), 161–183)
- Our approach: develop generic models and generic methods !

Goal of this talk: show what we do.

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   1. Simple Process Features
   2. More Complex Process Features
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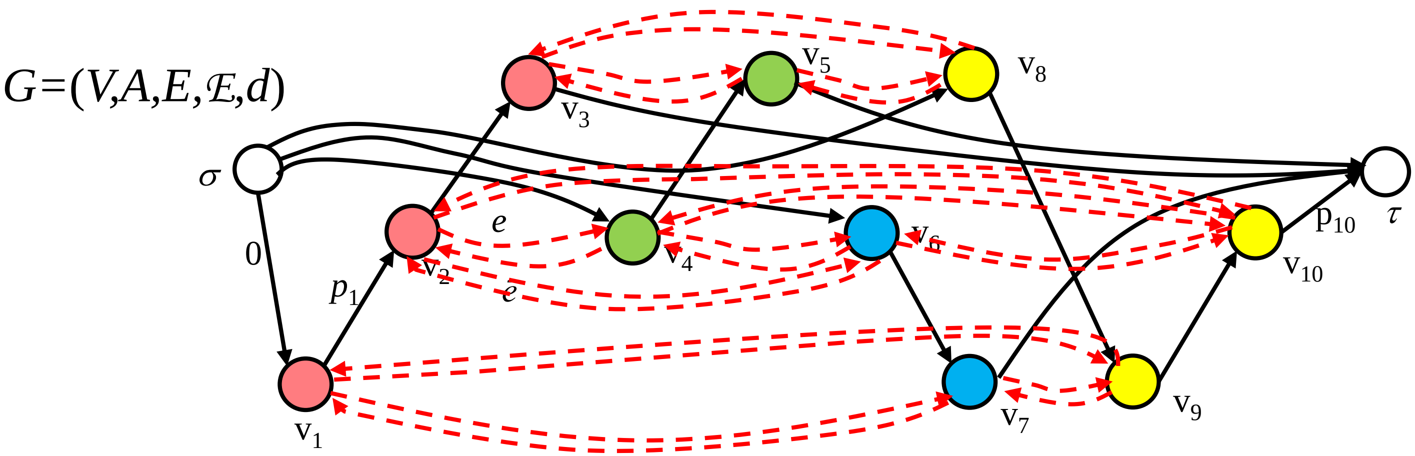
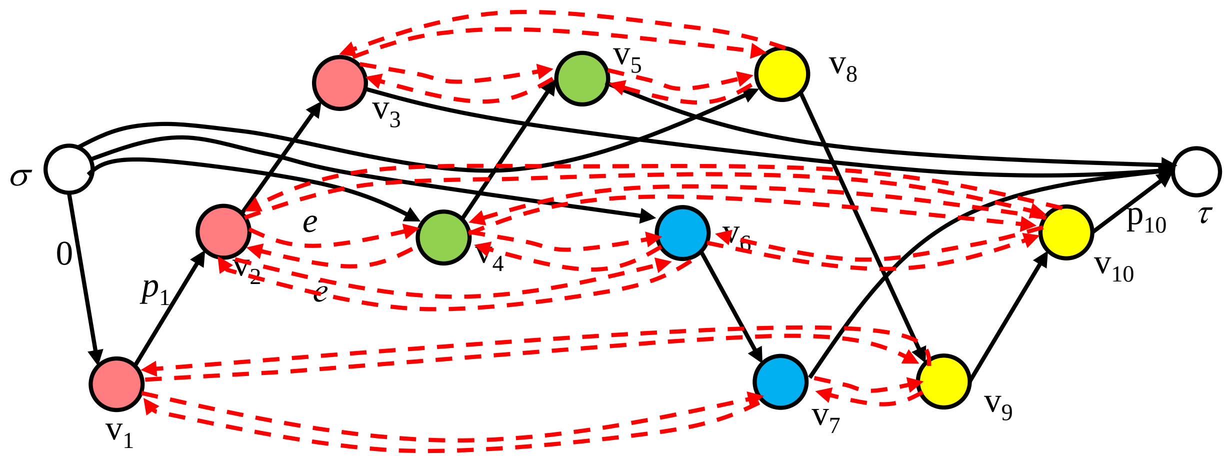
2. Complex Process Features

- Incorporate additional process features in our disjunctive graph formulation



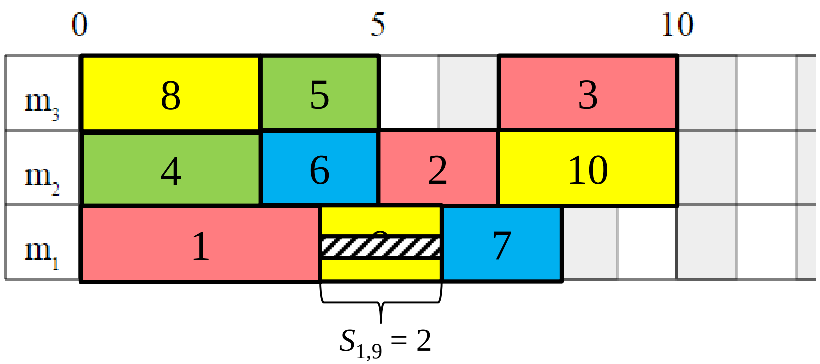
Among all feasible selections, find a selection S minimizing the length of a longest path from σ to τ in $G(S)$.

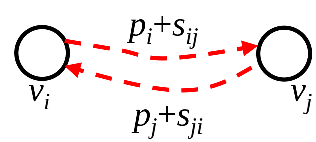


v5
v8
v3
σ
τ
e
v4
v6
p10
0
v2
v10
p1
e
v1
v7
v9

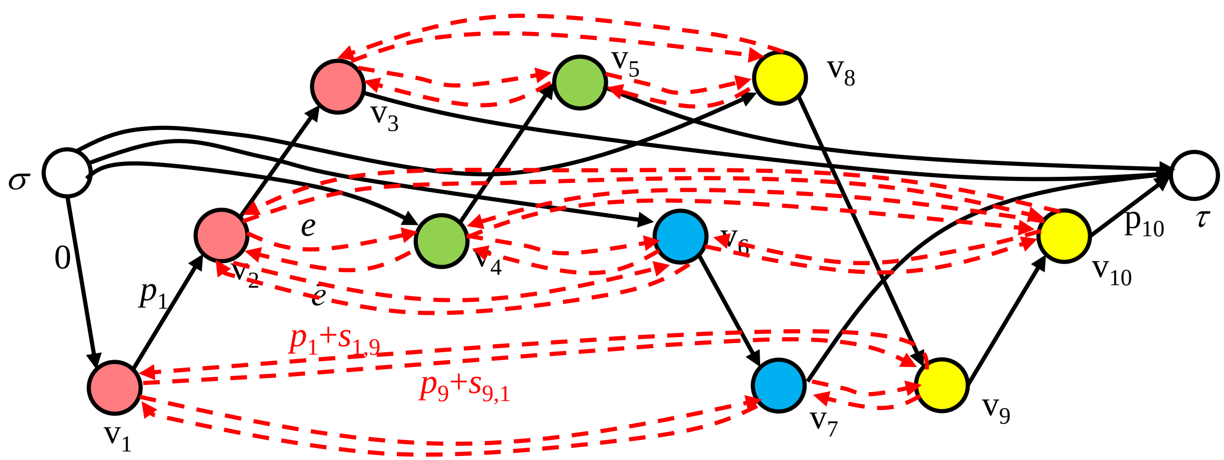
2.1. Some Process Features
Sequence-Dependent Setup Times

- If op. j directly follows op. i on some machine, then a setup of duration s_{ij} occurs between the completion of i and the start of j



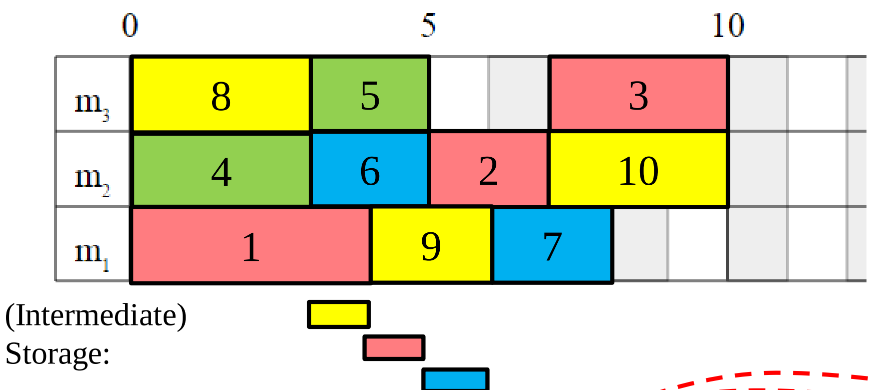


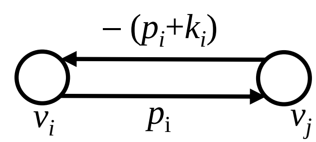
(Note: setups must satisfy so-called weak triangle inequalities.)



Limited Storage Time

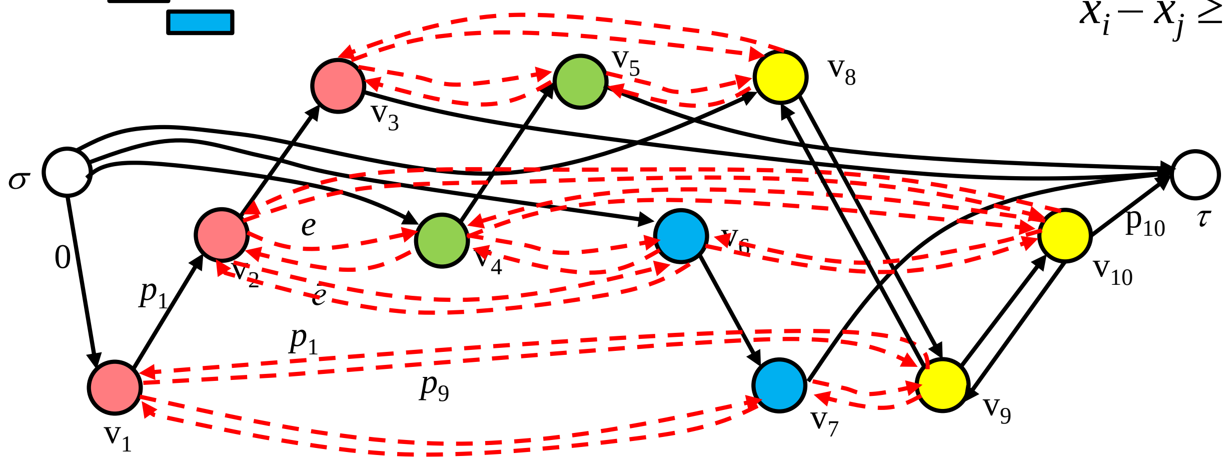
- After the completion of operation *i*, the job of *i* can be stored (or can wait) at most k_i time units before the processing of its next operation *j* starts (also called maximum time lag)





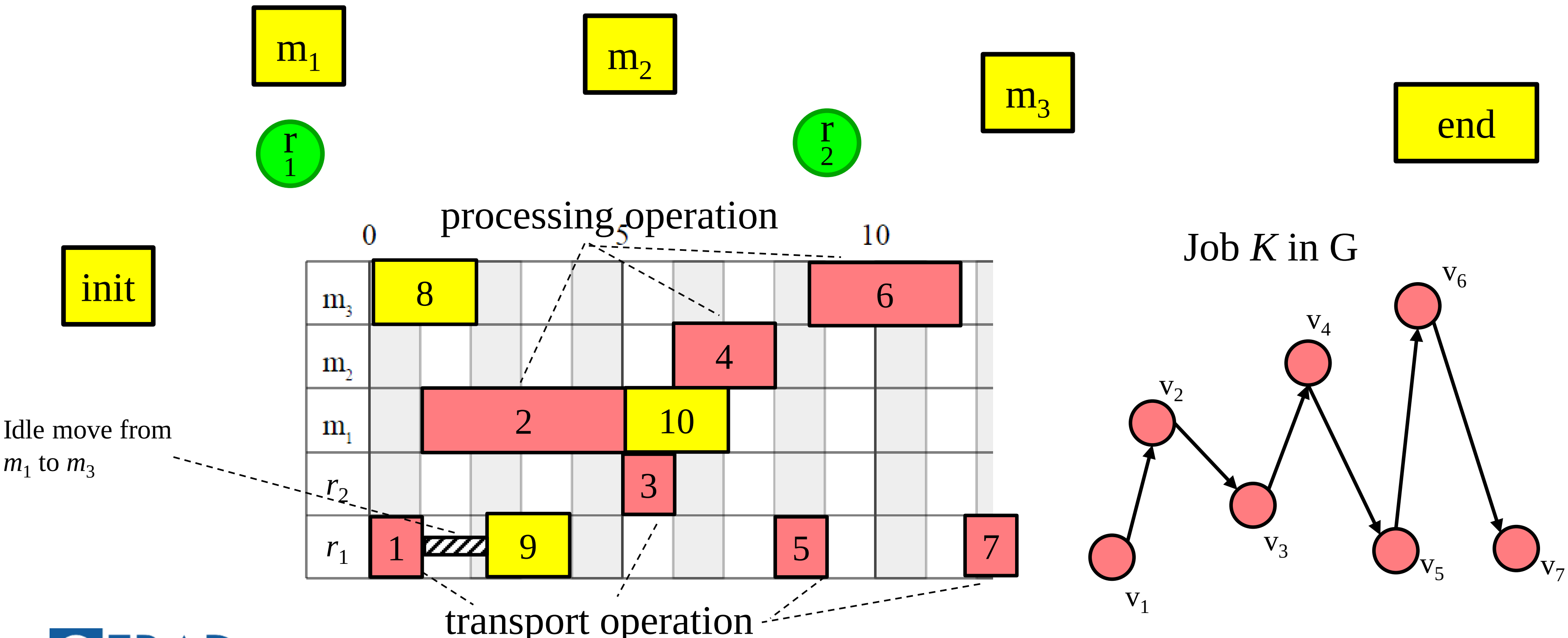
If $k_i = 0$ then it is a no-wait constraint

$$x_j - x_i \le p_i + k_i \iff$$
$$x_i - x_j \ge -(p_i + k_i)$$



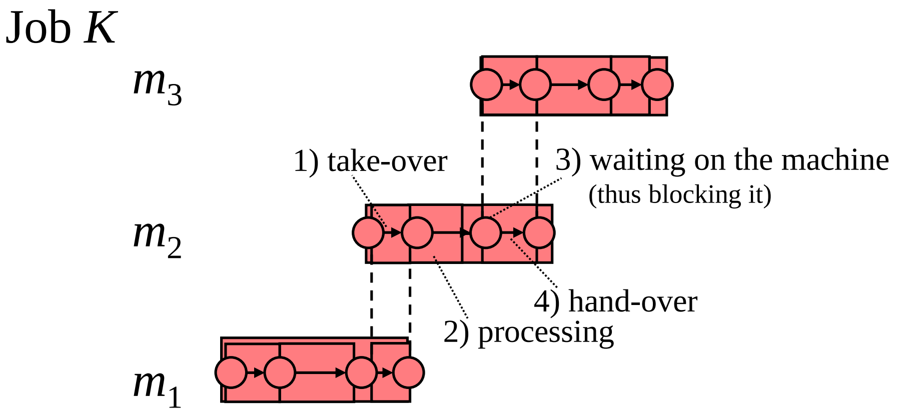
Transportation by Mobile Devices

- Mobile devices transport the jobs between the machines
- If the mobile devices can hold at most one job at any time, model them as machines
- Use sequence-dependent setup times for idle moves.



No (Intermediate) Storage Space - Blocking

- In the classical job shop, storage is not considered
  - Assumption: jobs can be stored somewhere
- However, often the storage space is limited or no (intermediate) storage space at all (e.g. robotic cells)
- Assume no storage space available, so called blocking.



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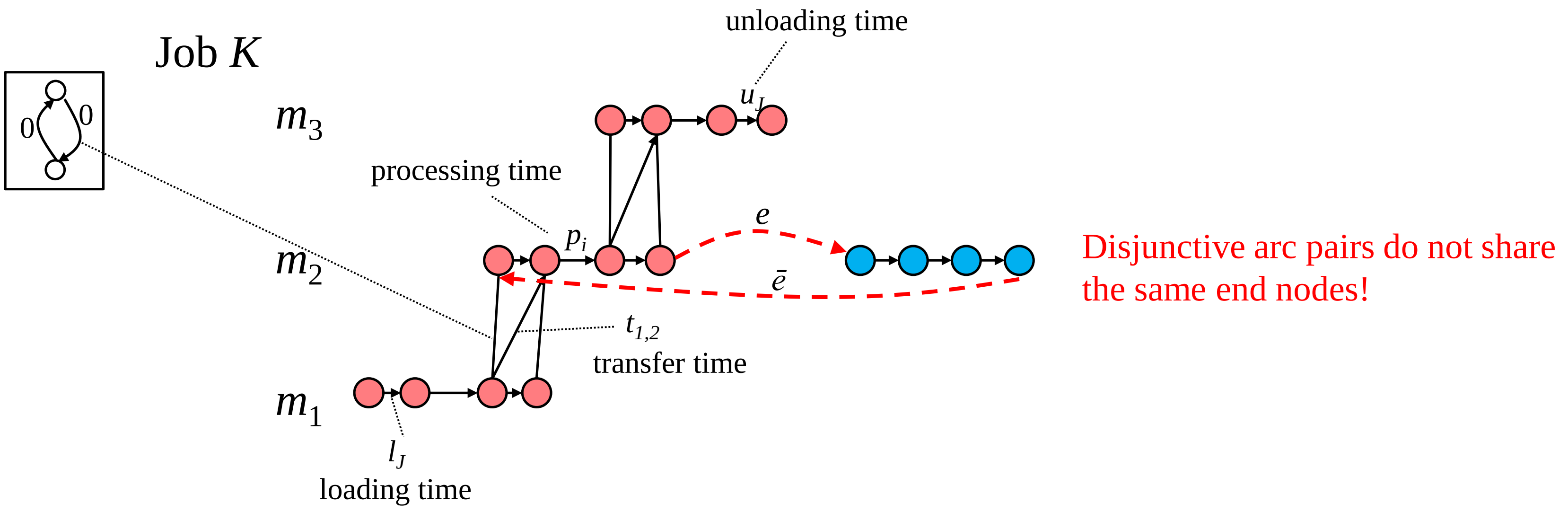
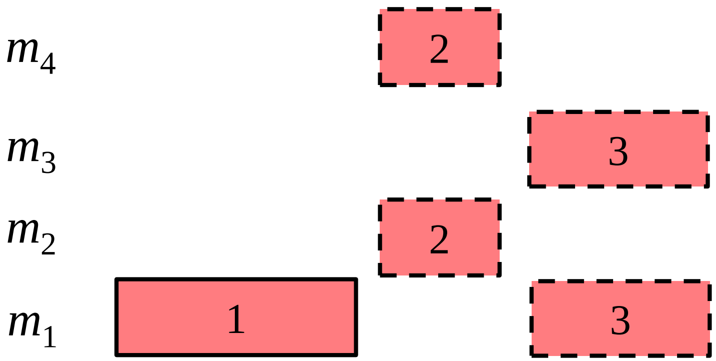
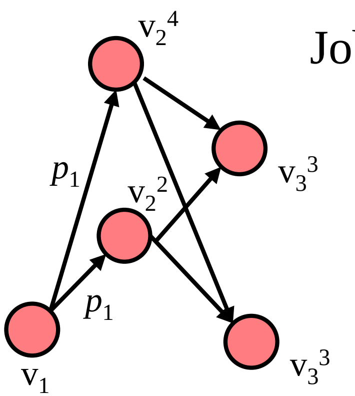


- Small number of storage units available: model them as machines

Routing Flexibility

- Different types of routing flexibility
  - Kis, T. (2003). Job-shop scheduling with processing alternatives. *European Journal of Operational Research*, *151*(2), 307–332
- Consider independent choice of the machine for each operation
  - For each operation i, the machine of i is not fixed but can be chosen from a subset of machines $M_i \subseteq M$
  - (Variable) Mode μ: choice of a machine $\mu_i \in M_i$ for each operation $i \in I$





Job K in G

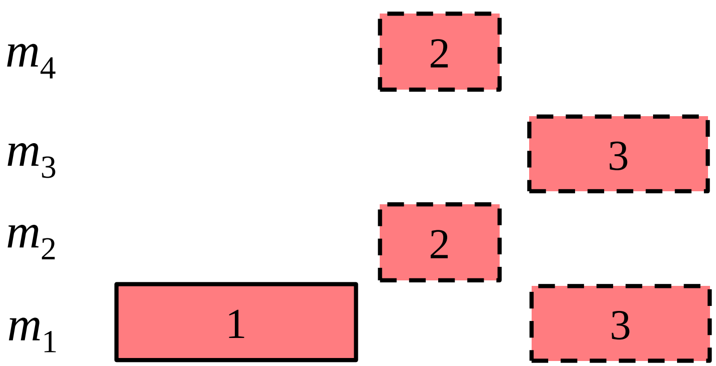
- With given mode μ, disjunctive graph $G^{\mu} = (V^{\mu}, A^{\mu}, E^{\mu}, \mathcal{E}^{\mu}, d)$
  (node-induced subgraph of G where nodes not belonging to mode μ are deleted)

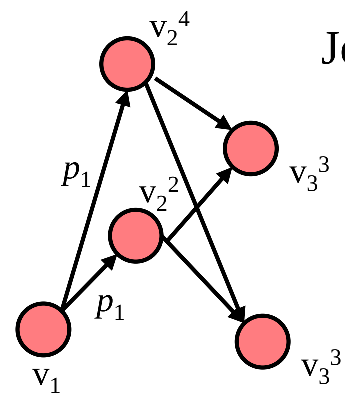
- Extended definition of selections:

Definition
A selection (μ,S): any mode μ and set $S \subseteq E$ of disjunctive arcs
A selection S is complete if $S \cap \{e,\bar{e}\} \neq \emptyset$ for all $\{e,\bar{e}\} \in \mathcal{E}^{\mu}$
A selection S is positive acyclic if the graph $G(\mu,S)=(V^{\mu},A^{\mu} \cup S,d)$ has no cycle of positive length and positive cyclic otherwise.

Among all feasible selections, find a selection (μ,S) minimizing the length of a longest path from σ to τ.

Gröflin, H., Pham, D. N., & Bürgy, R. (2011). The flexible blocking job shop with transfer and set-up times. *Journal of Combinatorial Optimization*, *22*(2), 121–144.

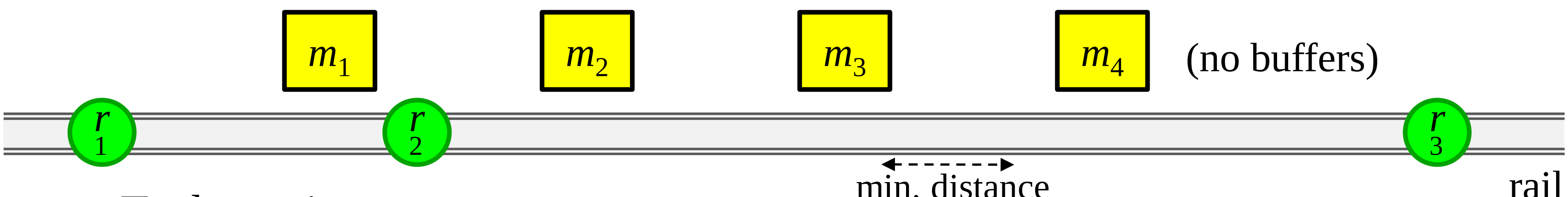




Job K in G

2.2. An Application: The BJS-RT and ALPHABOT
The BJS-RT

- Version of the blocking job shop (with routing flexibility) characterized by
- a rail-bound transportation system consisting of mobile devices (robots, cranes, …)
  - Processing of jobs on machines
  - Transport from one machine to the next by a robot which can be chosen
  - Robots move on a rail along which the machines are located
  - Robots cannot pass each other, maintain a minimal distance from each other
  - Each robot can handle at most one job at any time
  - Each robot can move at a speed up to a (robot-dependent) speed limit

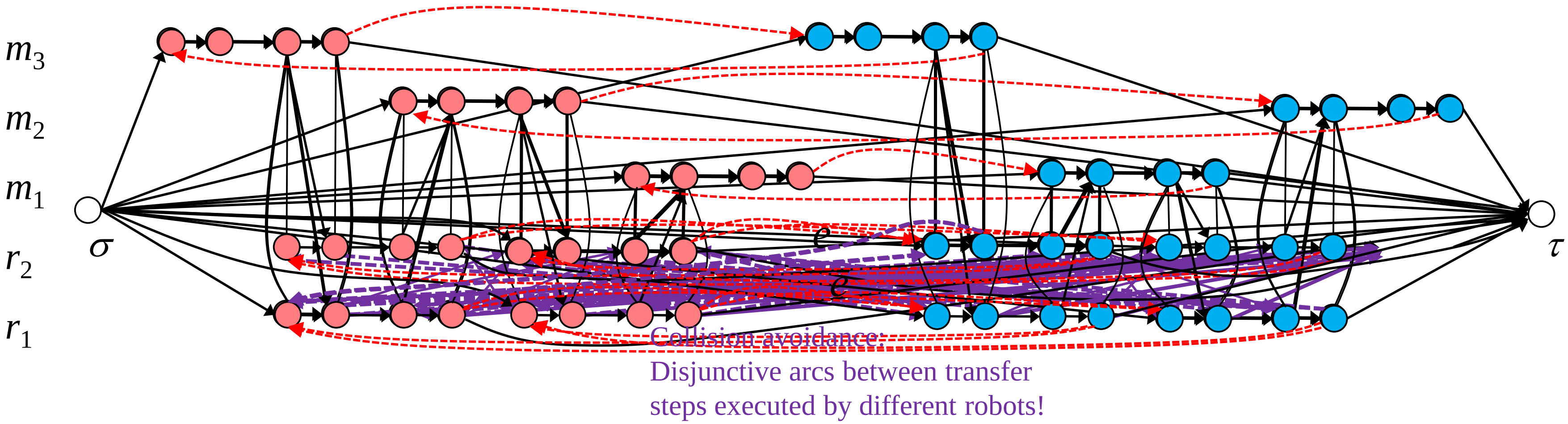


- To determine
  - Machine (robot) assignments and starting times (for processing and transport op.)
  - But also feasible trajectories of the robots

typical scheduling variables

No scheduling variables (and not in objective function)

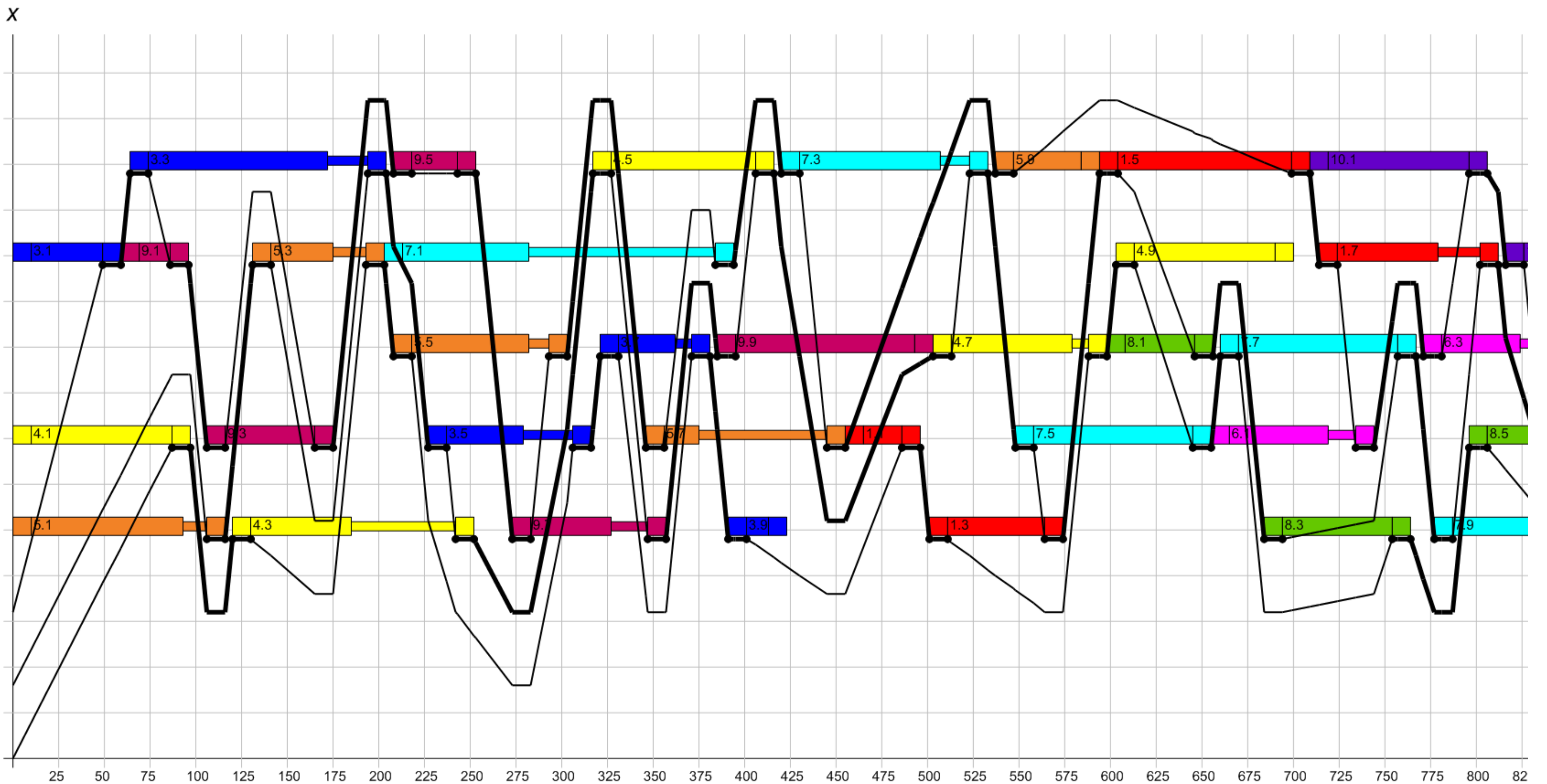
- Projection of the solution space onto the space of the assignment and time variables
  - Adding disjunctive arc pairs between *transfer steps* executed *by different robots*!



  - Yielding a formulation of the BJS-RT in a disjunctive graph
  - Allowing to apply our solution approach
  - Establish efficient algorithms for the feasible trajectory problem

See: Bürgy, R., & Gröflin, H. (2016). The blocking job shop with rail-bound transportation. *Journal of Combinatorial Optimization*, *31*(1), 151–181.

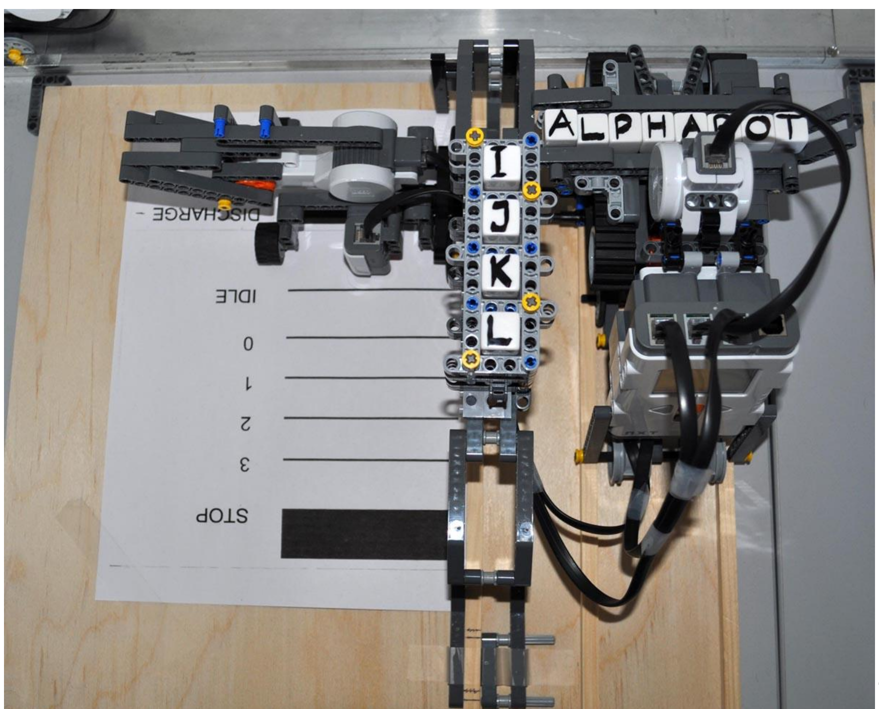
BJS-RT Schedules

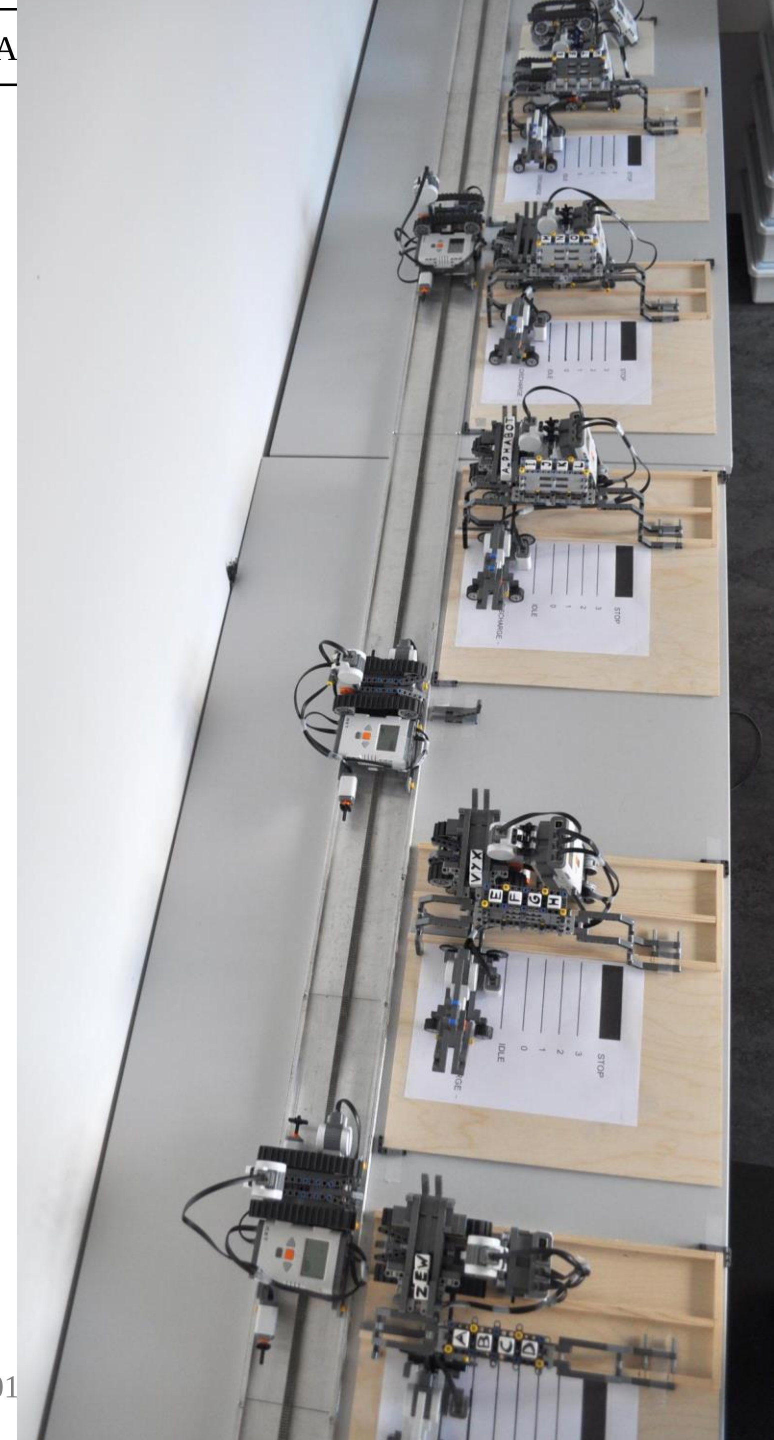


A “location-extended” Gantt chart

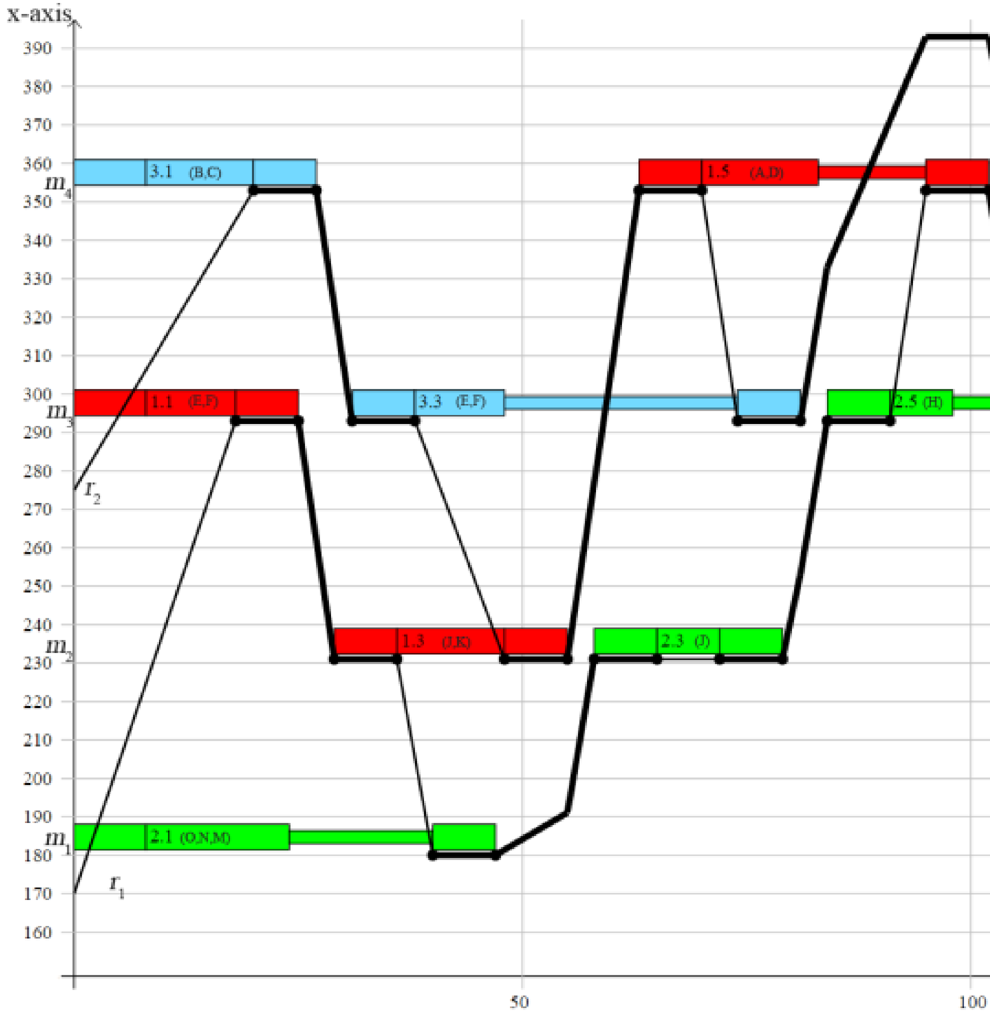
The ALPHABOT

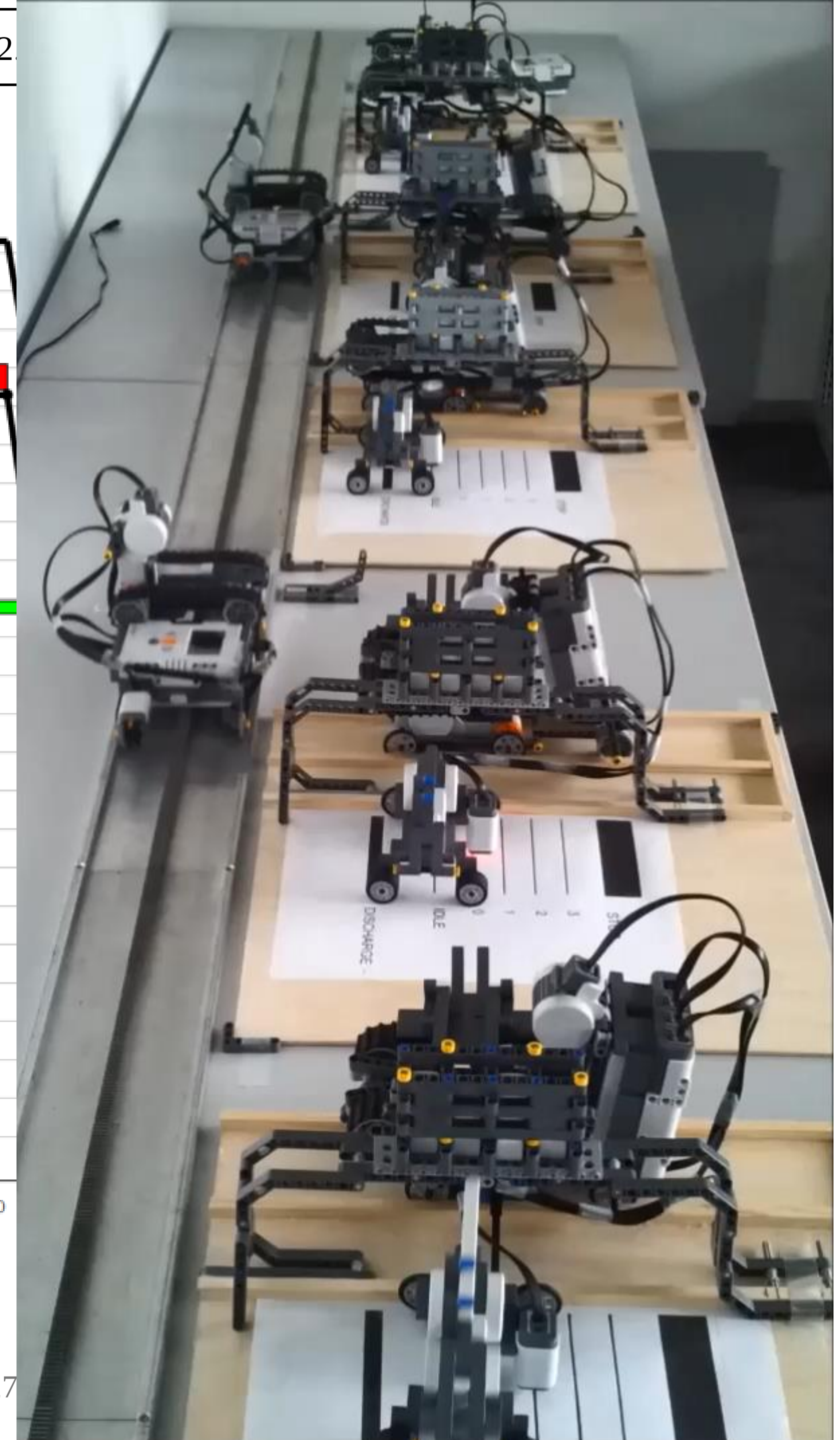
- A physical model of the BJS-RT
  - Machine: contains stacks of dices (with same letter)
  - Assemble words (a small container holds the dices)
  - Produce a given set of words as fast as possible





ALPHABOT in Action



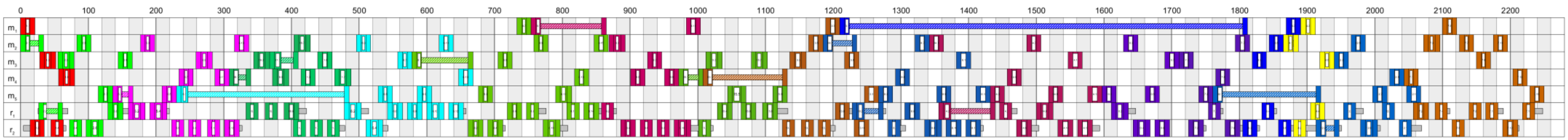


Robots and machines controlled by precedence constraints and NOT by (fixing) times!

Video: http://research.reinhardbuergy.ch/alphabot.php

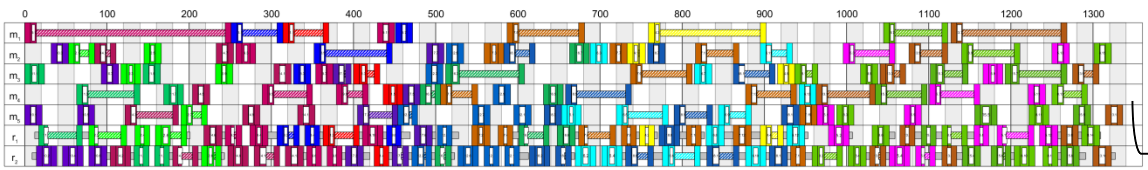
The Value of Optimization

- Example: Instance with 18 words (names)
  - Simple solution (job permutation schedule):



(trajectories omitted)

  - An optimized solution:



  - Nontrivial (even impossible) to find good plans by hand

improvement of 40%

- Some benefits
  - Quality improvement
  - (Semi-) automating complex work (decision support)
  - The role of the planner changes (addressing tactical questions)
  - Increased flexibility in the planning task

Overview

1. The Classical Job Shop Scheduling Problem
   1. Introduction
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2. Complex Process Features
   1. Some Process Features
   2. An Application: The BJS-RT and ALPHABOT
3. Complicated Objectives
   1. General Regular Objective
   2. A Class of Convex Cost Objectives
4. A Local Search Solution Approach
   1. The Job Insertion Problem
   2. Local Moves and Locally Improving Moves
   3. Some Computational Results

3. Complicated Objectives

- Consider again the timing problem in the classical job shop
- Timing subproblem:
  Given a selection S, solve:

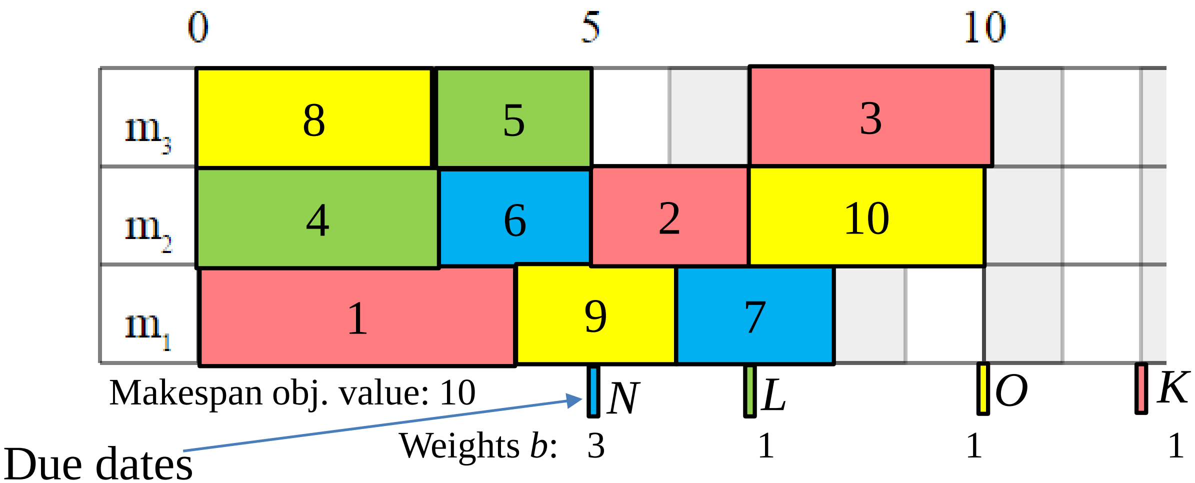
$$\begin{aligned}&\text{minimize} \quad \alpha_\tau - \alpha_\sigma \\ &\text{subject to}: \\ &\alpha_w - \alpha_v \geq d_{vw} \quad \text{for all } (v,w) \in A \cup S\end{aligned}$$

- Its dual, is a (simple) network flow problem.
- How to solve it: an optimal solution can be found by
  - Computing the earliest time schedule $\alpha(S)$ where
  - For all $v \in V$, $\alpha_v(S)$: length of a longest path from σ to node v in $G(S)$

3.1. General Regular Objective

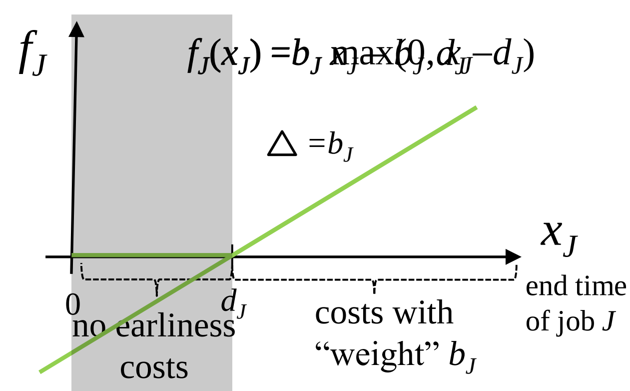
- It this procedure just applicable to the makespan objective?
- The class of regular objectives
  - "the earlier the better"
  - Formally, a function $f: \Re^V \rightarrow \Re$ is called regular if for all $\alpha, \alpha' \in \Re^V$, $\alpha \leq \alpha' \Rightarrow f(\alpha) \leq f(\alpha')$
  - Comprises many objectives: makespan, total flow time, total (weighted) tardiness, total squared tardiness, etc.
- The earliest time schedule $\alpha(S)$ is optimal
  - Let $\Omega(S)$ be the solution space of the timing problem
  - Clearly, $\alpha(S) \leq \alpha$ for all $\alpha \in \Omega(S)$, implying $f(\alpha(S)) \leq f(\alpha')$
  - Timing problem efficiently solvable!
  - Similar computational effort as for makespan objective
  - Somewhat higher for "re-optimization"
    - Consider: Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2015). Timing Problems and Algorithms: Time Decisions for Sequences of Activities. Networks, 65(2), 102–128.

An Example with Tardiness Costs

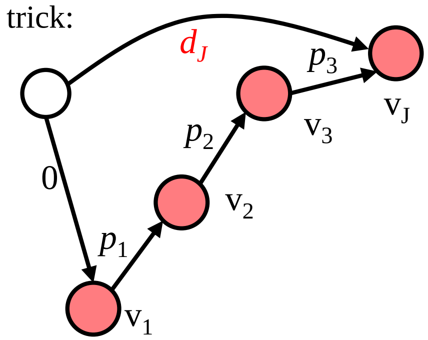


Total linear tardiness costs: 9 (job N 3 time units too late)

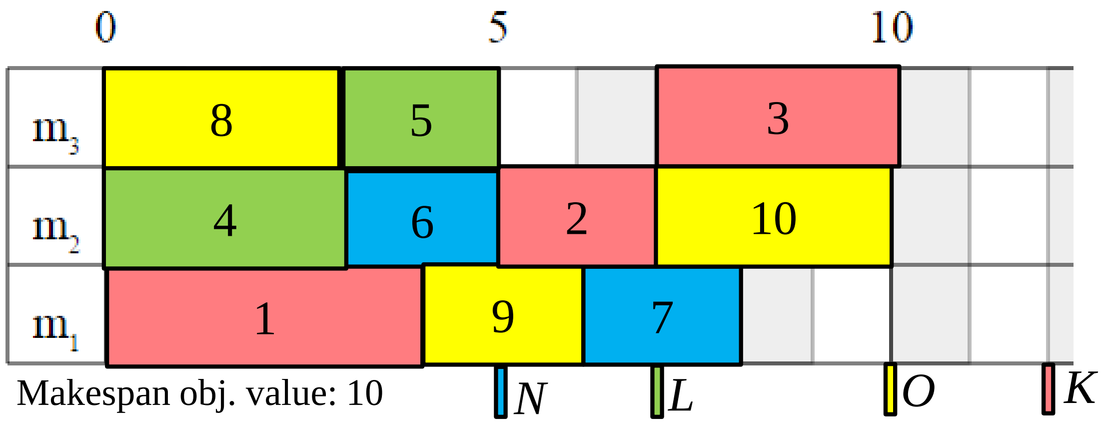
Linear tardiness costs (for each job J)



Note: non-linear function in x_J, simple "linearization" modeling trick:

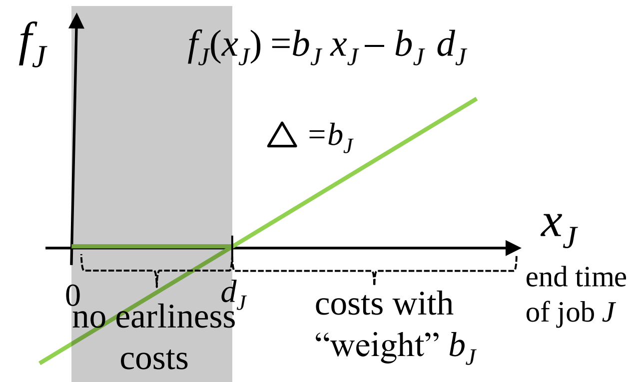


An Example with Tardiness Costs



Weights b: 3 1 1 1

Linear tardiness costs (for each job J)



Total linear tardiness costs: 9 (job N 3 time units too late)

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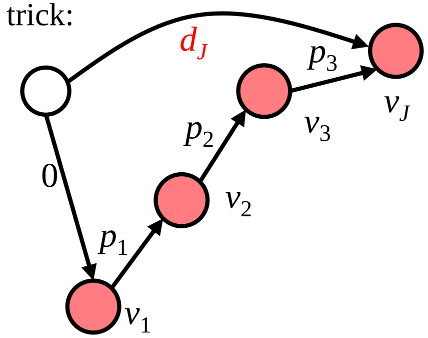
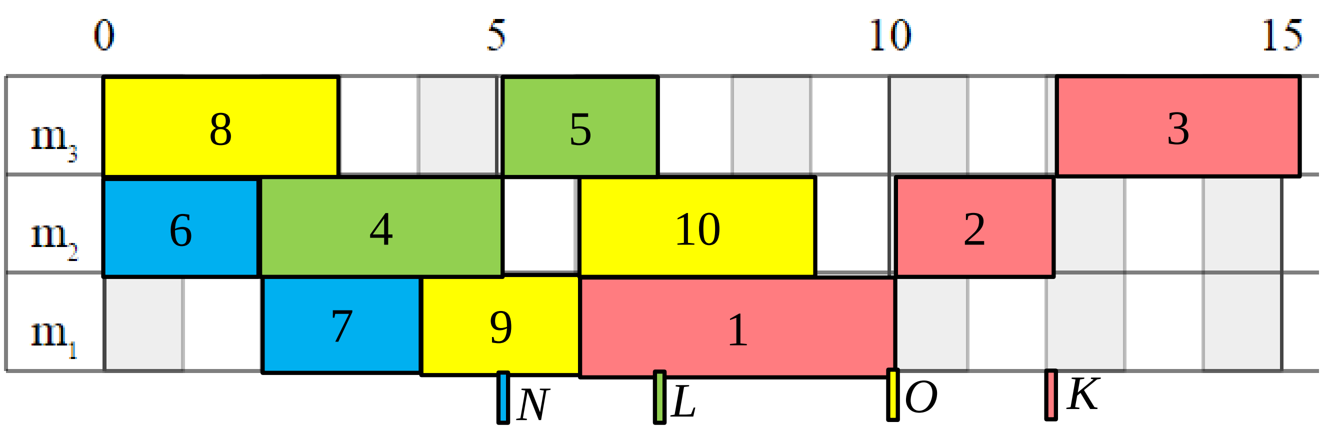
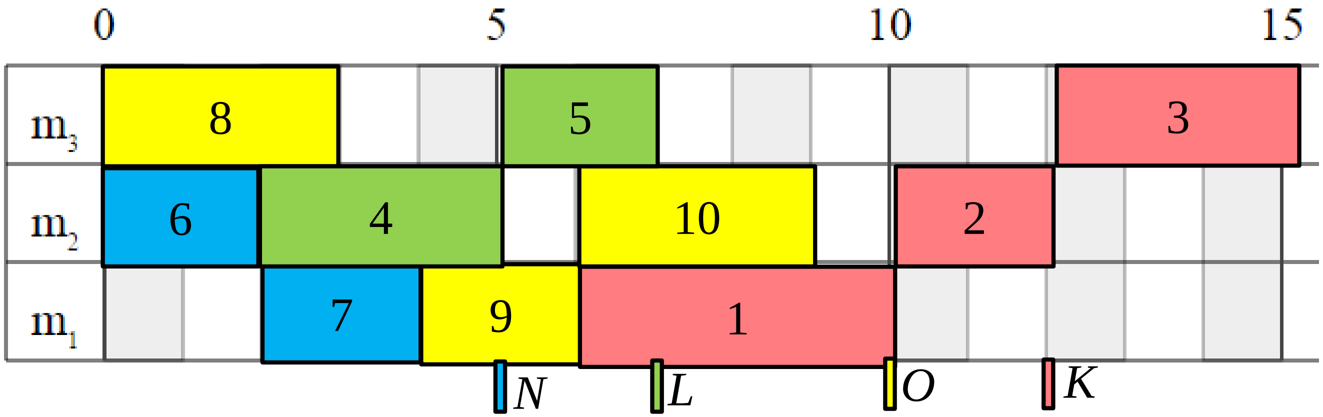


With improved sequencing:



Total linear tardiness costs: 3 (job K 3 time units too late)

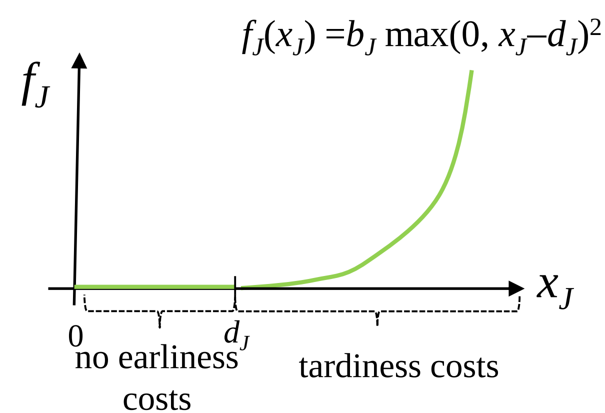


Total linear tardiness costs: 3 (job K 3 time units too late)

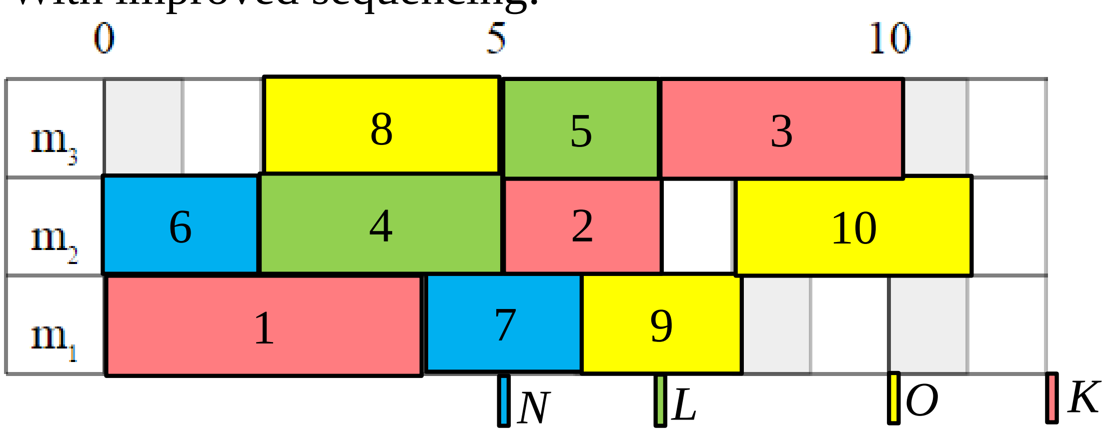
Total squared tardiness costs: 9 (job K 3 time units too late)

However, a large tardiness may be very undesirable in practice → squared tardiness costs:

Squared tardiness costs



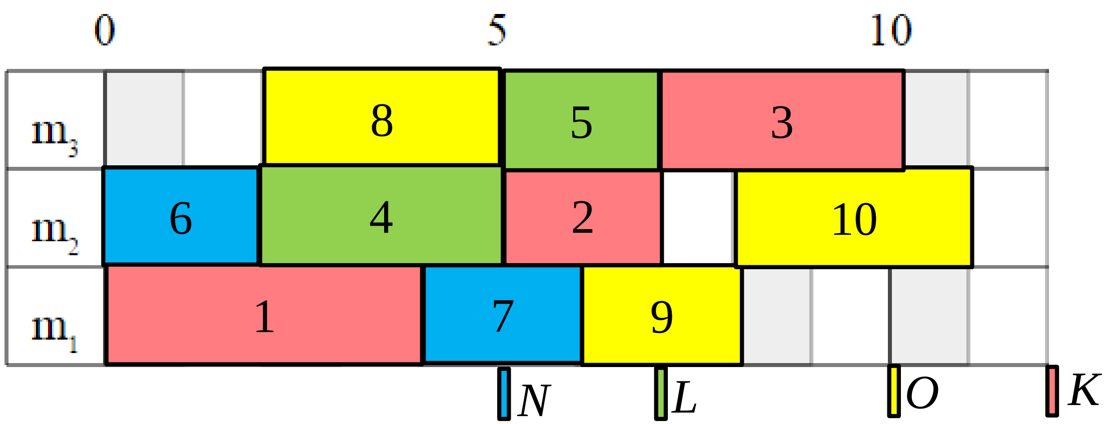
With improved sequencing:

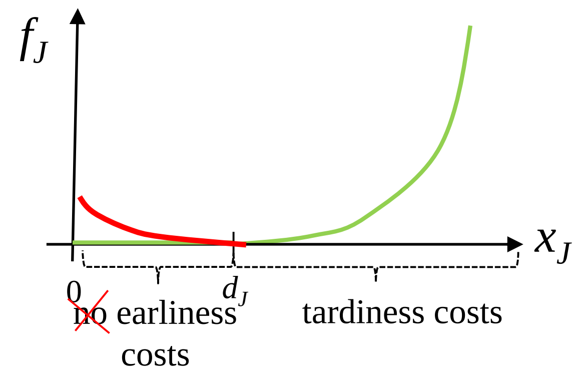


still regular objective!

Total squared tardiness costs: 4 (Job O 1 and Job N 1 time unit too late)

3.2. A Class of Convex Cost Objectives

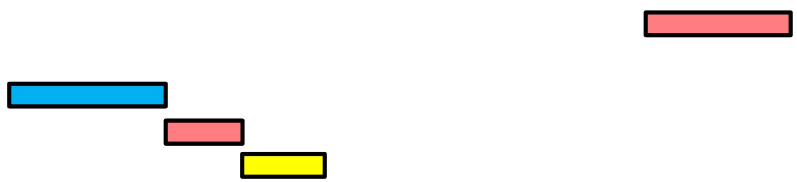




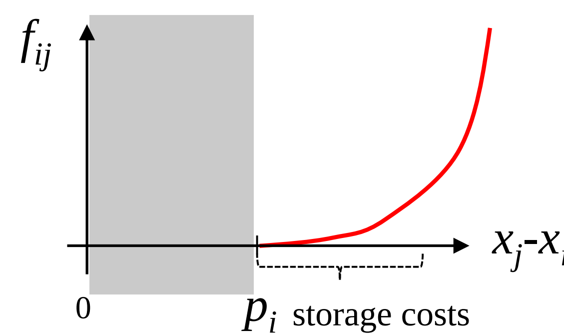
Total squared tardiness costs: 4 (Job *O* 1 and Job *N* 1 time unit too late)

Earliness:

Storage:



Op. *i* and *j* consecutive in some job



Just-in-time scheduling:
Take into account convex tardiness, earliness and storage costs!

Total Convex Costs: The Timing Problem

Makespan objective:

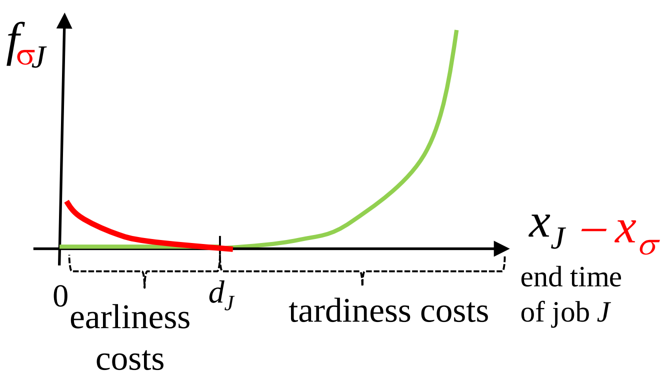
$$\begin{aligned}&\text{minimize} \quad \alpha_\tau - \alpha_\sigma\\&\text{subject to:}\\&\alpha_w - \alpha_v \geq d_{vw} \quad \text{for all } (v,w) \in A \cup S\end{aligned}$$

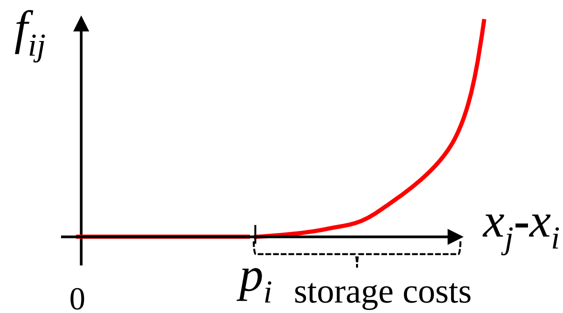
A total convex costs objective:

$$\begin{aligned}&\boxed{\text{minimize} \quad \sum_{(v,w)\in F} f_{vw}(\alpha_w - \alpha_v)} \quad \boxed{f_{vw} \text{ convex}}\\&\text{subject to:}\\&\alpha_w - \alpha_v \geq d_{vw} \quad \text{for all } (v,w) \in A \cup S\end{aligned}$$

"Precision" of our plan.
(implied with regular objective and integer data)

U: "large" planning horizon
(for technical reasons)





A Solution Approach for This Timing Problem

f_{vw} convex

$$\text{minimize} \quad \sum_{(v,w)\in F} f_{vw}(\alpha_w - \alpha_v)$$

subject to:

$$\alpha_w - \alpha_v \ge d_{vw} \quad \text{for all } (v,w) \in A \cup S$$

$$\alpha_v \text{ integer for all } v \in V$$

$$0 \le \alpha_v \le U \quad \text{for all } v \in V$$

- Still a network flow problem?
- A convex cost integer dual network flow problem!

Ahuja, R. K., Hochbaum, D. S., & Orlin, J. B. (2003). Solving the Convex Cost Integer Dual Network Flow Problem. *Management Science*, *49*, 950–964

  - Show that the Lagrangian relaxation of the problem (actually a reformulation) can be transformed to a network flow problem with (special) convex costs
  - Adapt the cost-scaling algorithm for the minimum cost flow problem to solve the convex cost network flow problem (obtaining also an optimal dual solution)
  - Overall time complexity: $O(nm \log(n^2/m) \log(nU))$

also shown by:

- Hence, timing problem still efficiently solvable!
  - Higher time complexity than for regular objectives

Stephan Foldes and François Soumis. PERT and crashing revisited: Mathematical generalizations. *European Journal of Operational Research* 64.2 (1993): 286-294.

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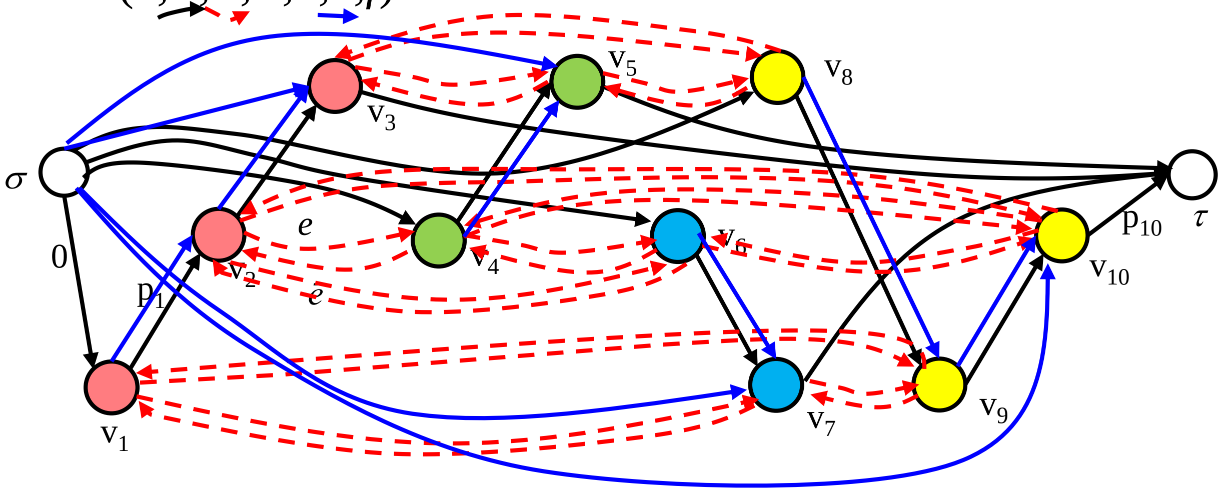
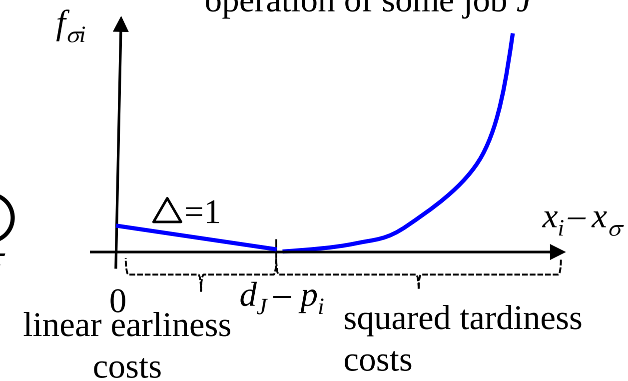
- The job shop scheduling problem with convex costs:

$G=(V,A,E,\mathcal{E},d,F,f)$



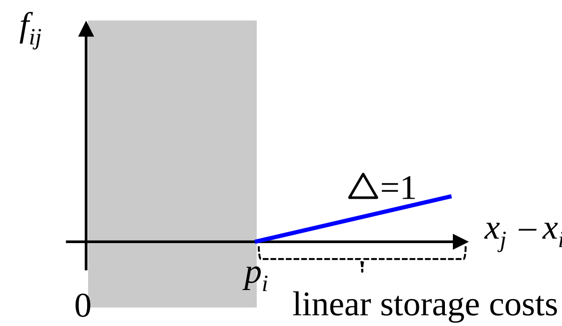
Op. i being the last operation of some job J



$f_{\sigma i}(x_i - x_\sigma) = \max(0, (d_J - p_i) - (x_i - x_\sigma)) + \max(0, (x_i - x_\sigma) - (d_J - p_i))^2$

Just-in-time job shop scheduling with squared tardiness costs and linear storage costs

Op. i and j consecutive in some job

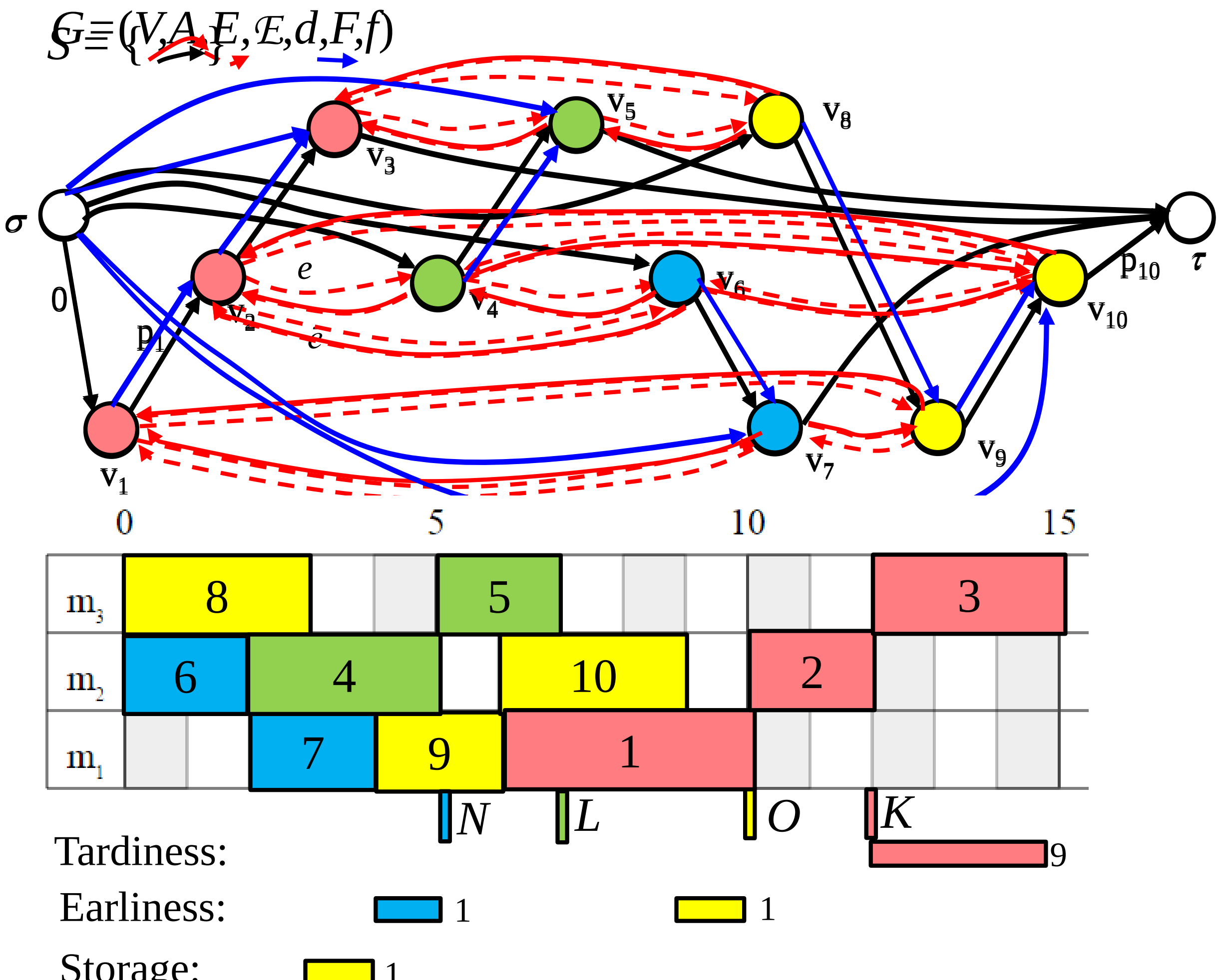


Among all feasible selections, find a selection S with minimum total convex costs.

$f_{\sigma i}(x_j - x_i) = x_j - x_i - p_i$

General Scheme

- Local search based on a job insertion neighborhood



Total costs: 12

Neighbor generation: Extract a job and re-insert it into the given schedule

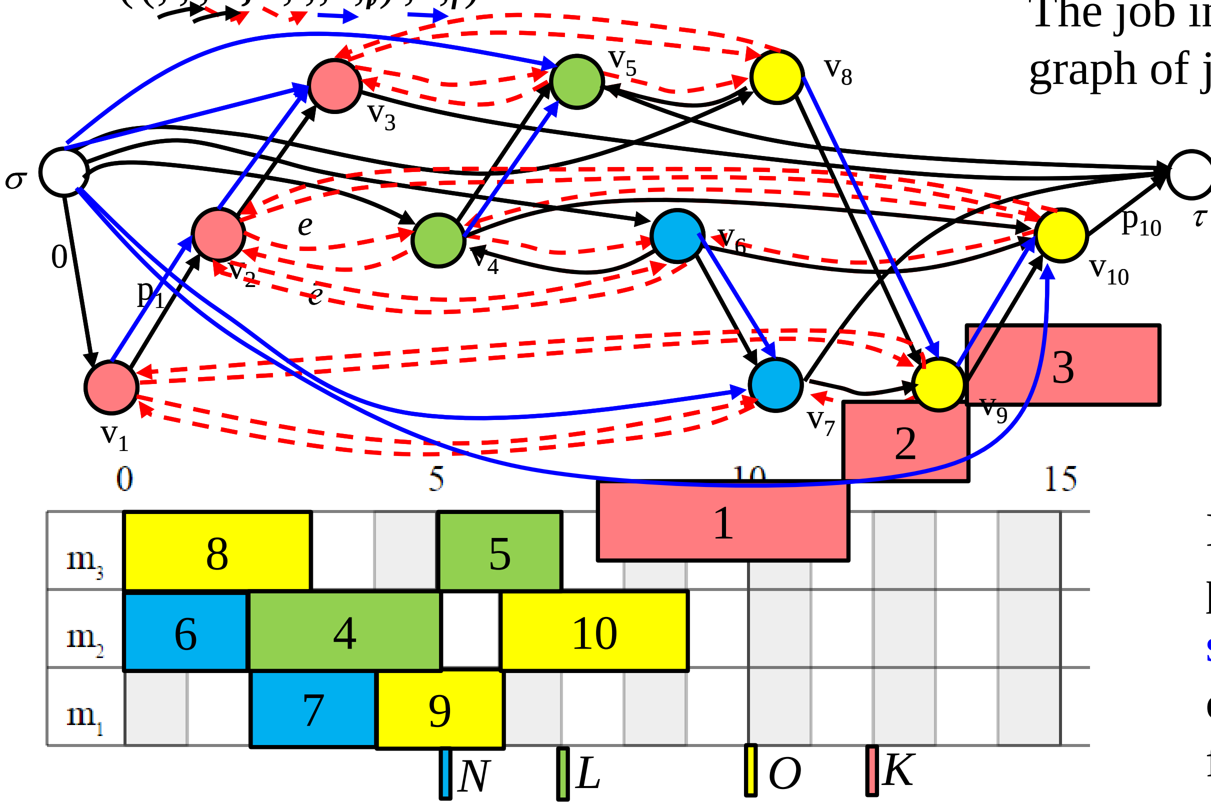
„Given schedule“: fixing disjunctive arcs, not starting times!

See:
Gröflin, H., & Klinkert, A. (2007). Feasible insertions in job shop scheduling, short cycles and stable sets. *European Journal of Operational Research*, *177*(2)

4.1. The Job Insertion Problem

- Problem formulation in its associated disjunctive graph:

$G^K=(V,A^K,E^K,d,F^K,f)$

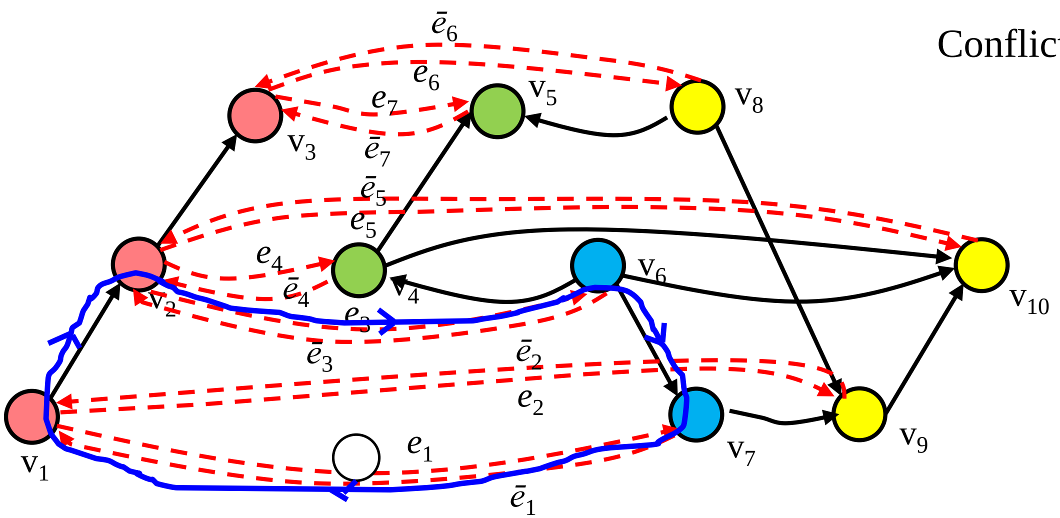


The job insertion disjunctive graph of job *K*.

In this graph, (complete, positive acyclic, feasible) selections (complete are called positive acyclic, feasible) insertions

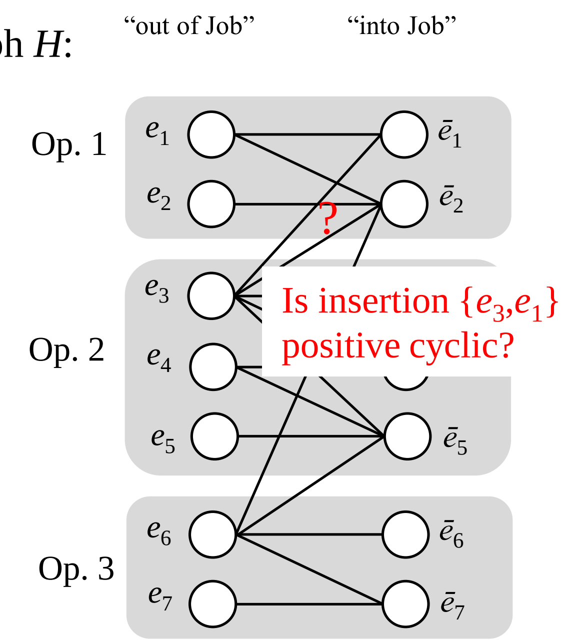
The Conflict Graph

- Characterize ALL feasible insertions in an associated graph



(nodes σ, τ and arcs of F omitted)

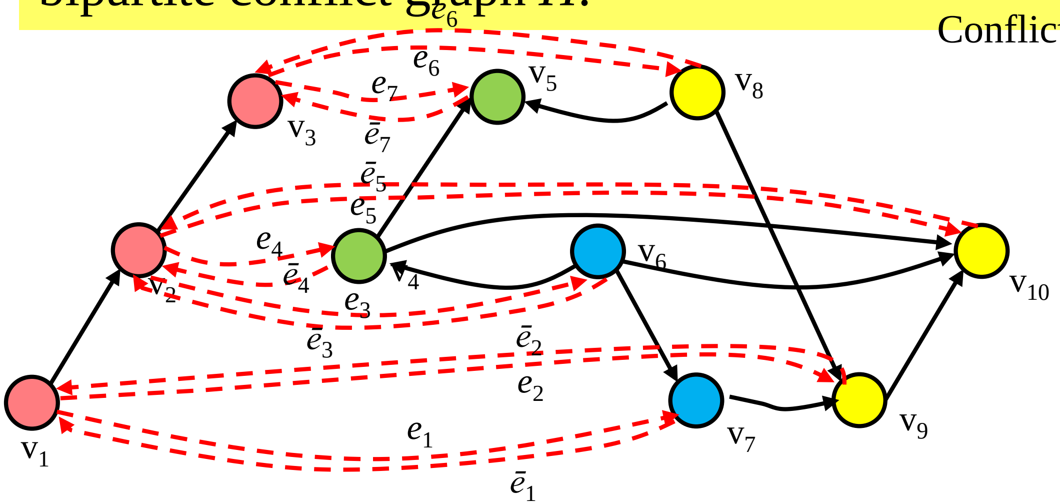
Conflict Graph H:



Definition
Given a job insertion graph $G^J = (V^J,A^J,E^J,\mathcal{E}^J,d,F,f)$, the conflict graph of G^J is the undirected graph $H= (E^J,U)$ where for any $e,f \in E^J$, $(e,f\,) \in U$ if insertion $\{e,f\}$ is positive cyclic.

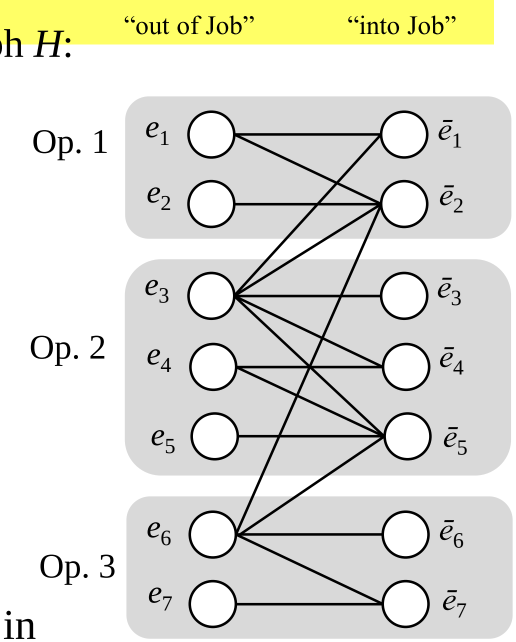
Theorem

Given a job insertion graph $G^J = (V^J, A^J, E^J, \mathcal{E}^J, d)$, the feasible insertions are in one-to-one correspondence with the stable sets of size $|E^J|/2$ in the bipartite conflict graph H.



(nodes σ, τ and arcs of F omitted)

Conflict Graph H:

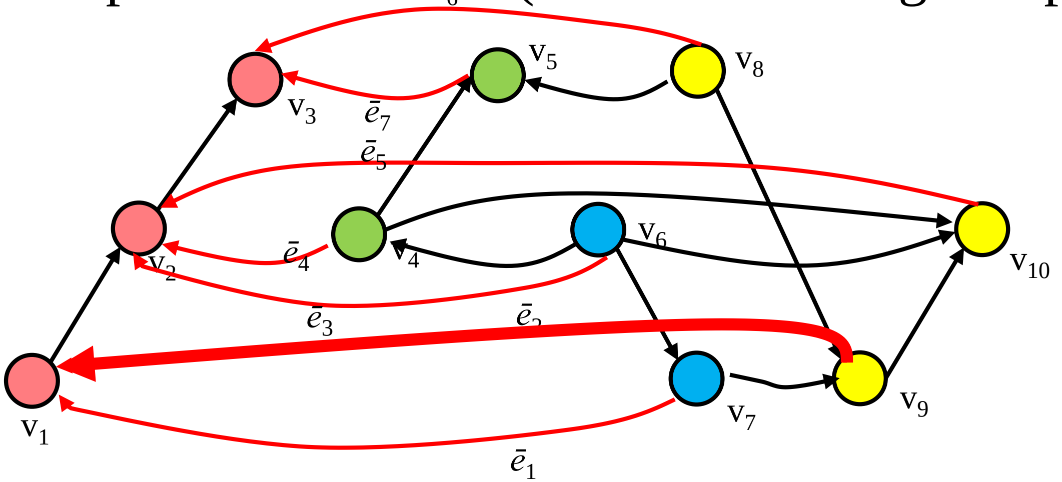


Proof, see: Gröflin, H., & Klinkert, A. (2007). Feasible insertions in job shop scheduling, short cycles and stable sets. *European Journal of Operational Research*, *177*(2)

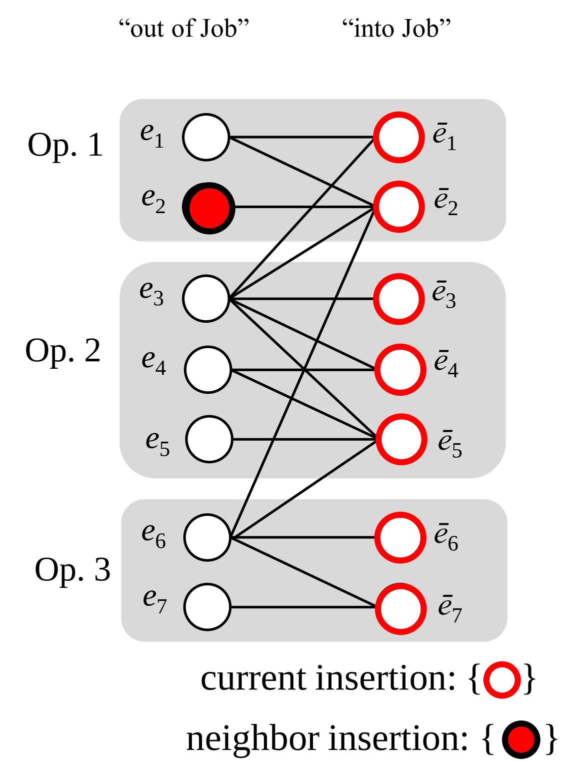
- $\Rightarrow$ Nice (polyhedral) characterization of all feasible insertions
- Generate neighbor insertions in the conflict graph.

4.2. Local Moves and Locally Improving Moves

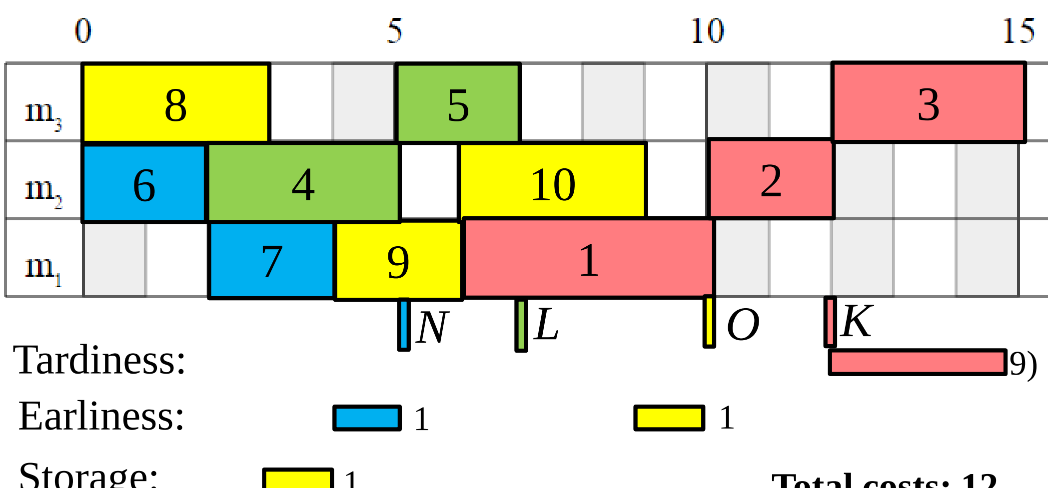
- Replace a “critical” disjunctive arc, i.e. a disjunctive arc with positive arc flow (in dual of timing sub-problem), by its mate



Conflict Graph H:



Arc flow of $\bar{e}_2$ is 2. Replace $\bar{e}_2$ by e_2.



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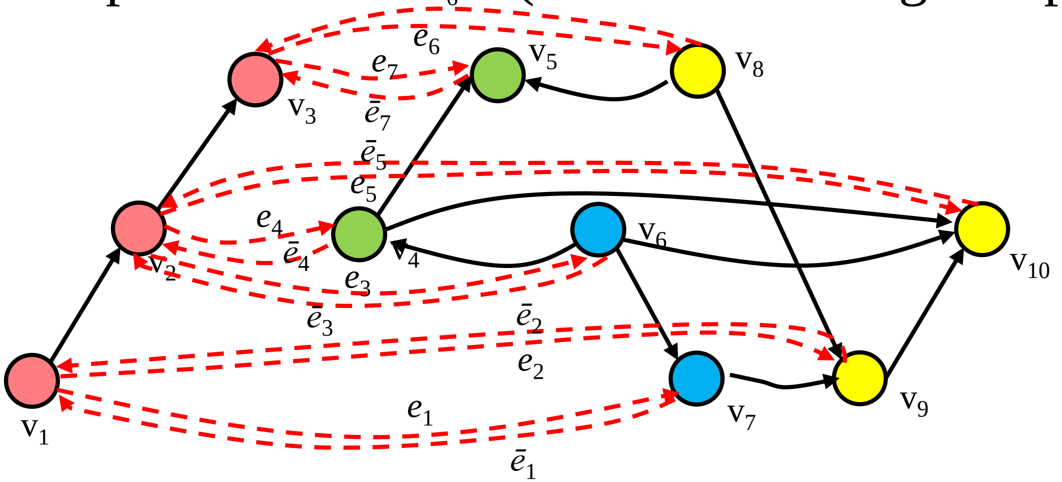
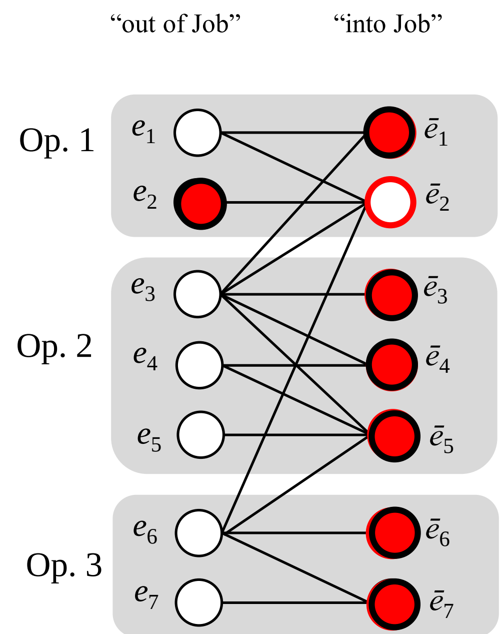


Conflict Graph H:



current insertion: { }

neighbor insertion: { }

Arc flow of $\bar{e}_2$ is 2. Replace $\bar{e}_2$ by e_2.

- Generate nearest insertion:

$$T_{\bar{e}} = \bar{e} \cup (T^S \setminus \{e\})$$

"Swapping" a critical arc!

Proposition
$T_{\bar{e}}$ is a feasible insertion.

See e.g. Brandimarte, P., & Maiocco, M. (1999). Job shop scheduling with a non-regular objective: A comparison of neighbourhood structures based on a sequencing/timing decomposition. *International Journal of Production Research*, *37*(8), 1697–1715

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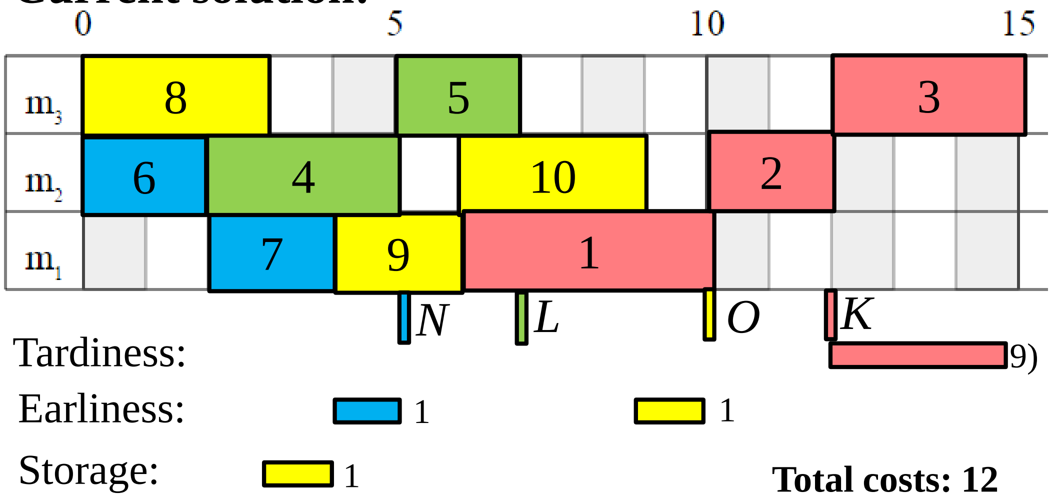
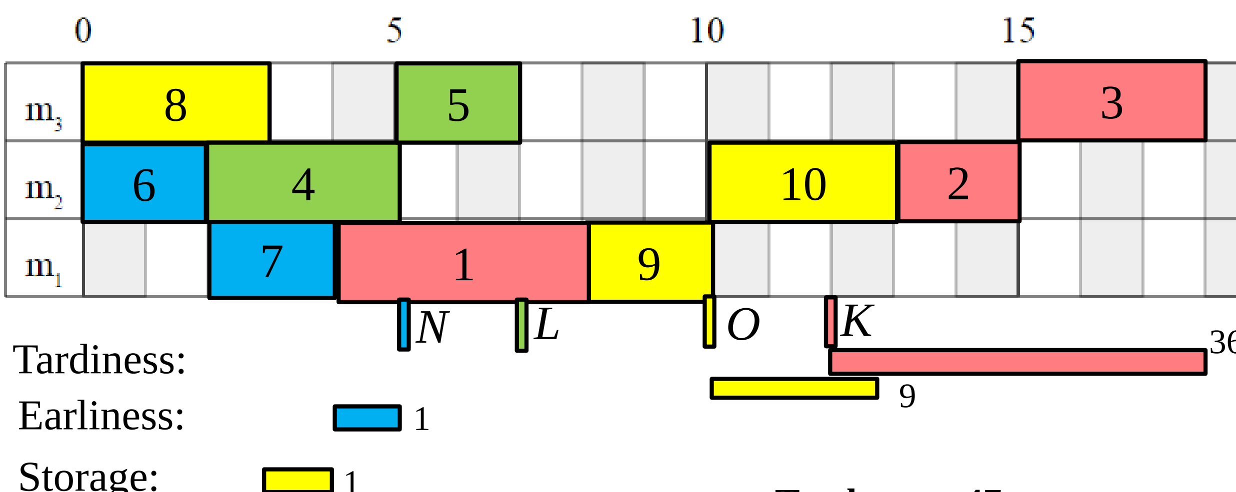
Current solution:



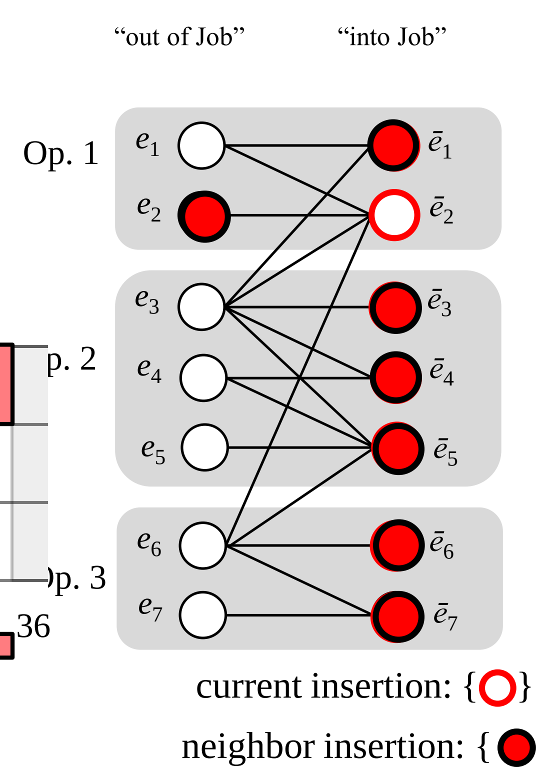
Total costs: 12

Neighbor solution:



Total costs: 47

Conflict Graph H:



Optimal Job Insertion

This problem is NP-hard already in the classical job shop.

- Use conflict graph and its associated MIP formulation to compute an optimal job insertion

Opt. Job Insertion Algorithm

1. Solve MIP
2. *IF* MIP feasible *DO*
3. compute optimal times,
4. and store solution if best,
5. forbid current insertion
6. Go To 1.
7. *ELSE* stop

weights (see later)

$$\text{minimize} \quad \sum_{v \in E^J} c_v x_v$$

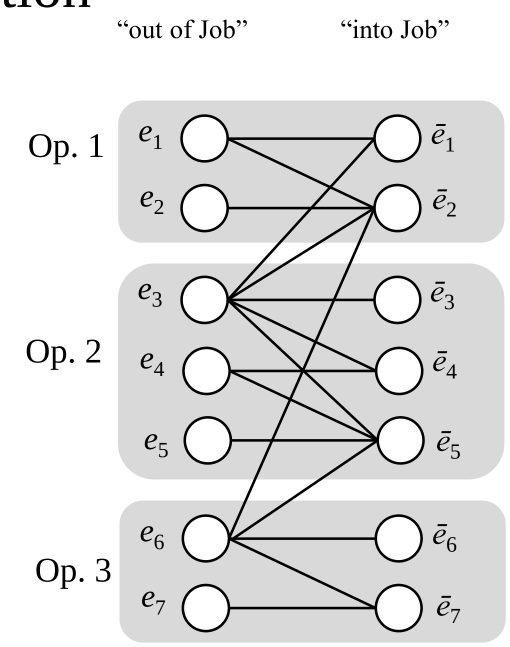
$$\text{subject to:}$$

$$x_w + x_v \le 1 \quad \text{for all } \{v,w\} \in U$$

$$x_v + x_w = 1 \quad \text{for all } \{v,w\} \in \mathcal{E}^J$$

$$x_v \in \{0,1\} \quad \text{for all } v \in E^J$$

Conflict Graph $H = (E^J, U)$:



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6. Go To 1.
7. *ELSE* stop

$$\text{minimize} \quad \sum_{v \in E^J} c_v x_v$$

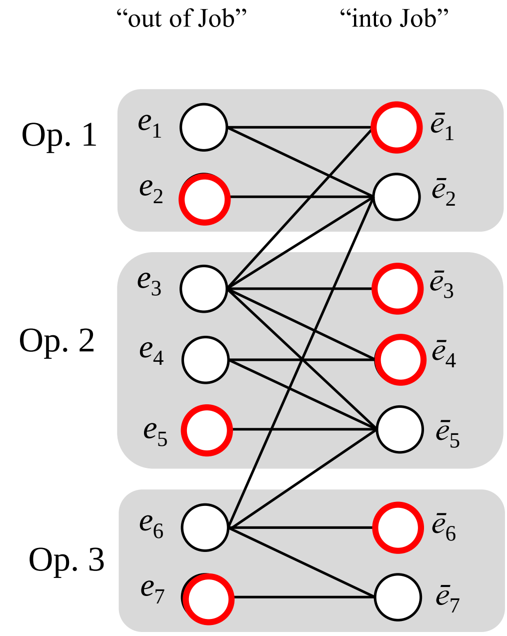
subject to:

$$x_w + x_v \le 1 \quad \text{for all } \{v,w\} \in U$$

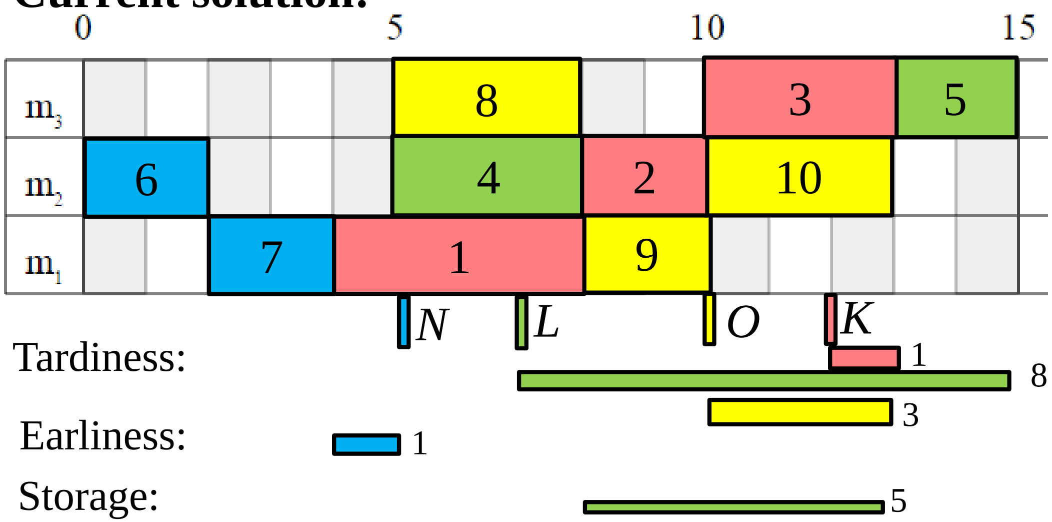
$$x_v + x_w = 1 \quad \text{for all } \{v,w\} \in \mathcal{E}^J$$

$$x_v \in \{0,1\} \quad \text{for all } v \in E^J$$

Conflict Graph $H = (E^J, U)$:



Current solution:



Total costs: 80

Forbid current insertion $I = \{\bigcirc\}$:
(simple way) add constraint:

$$\sum_{v \in I} x_v \le |I| - 1$$

However, then ALL feasible insertions are enumerated! (there are already 25 in the small example)

Opt. Job Insertion Algorithm

1. Solve MIP
2. *IF* MIP feasible *DO*
3. compute optimal times,
4. and store solution if best,
5. forbid current insertion
6. Go To 1.
7. *ELSE* stop

$$\text{minimize} \quad \sum_{v \in E^J} c_v x_v$$

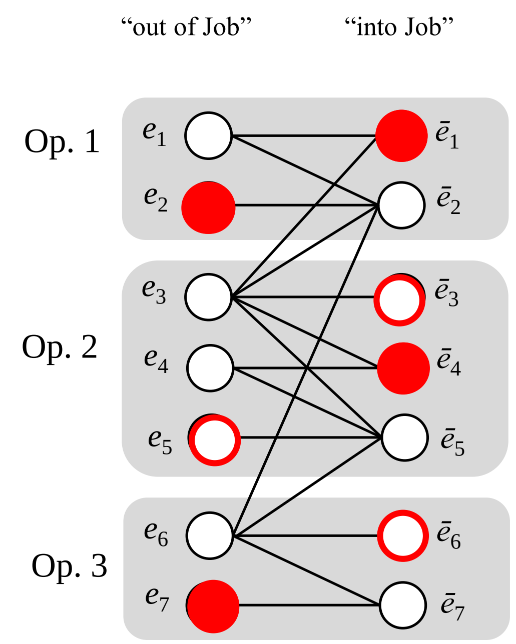
subject to:

$$x_w + x_v \leq 1 \quad \text{for all } \{v,w\} \in U$$

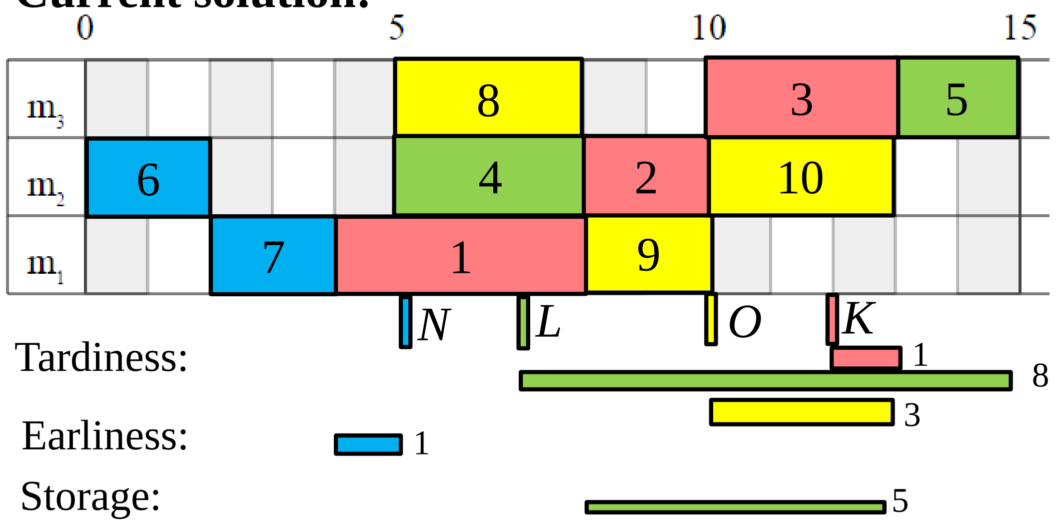
$$x_v + x_w = 1 \quad \text{for all } \{v,w\} \in \mathcal{E}^J$$

$$x_v \in \{0,1\} \quad \text{for all } v \in E^J$$

Conflict Graph $H = (E^J, U)$:



Current solution:



Total costs: 80

Forbid current insertion $I = \{\bigcirc\}$:

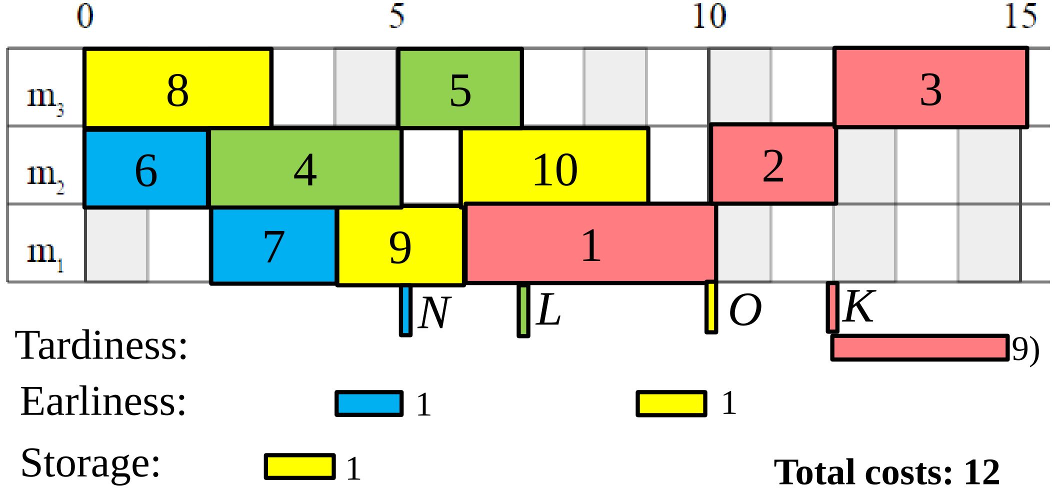
Better: just forbid critical arc set!

$I^{\text{crit}} = \{\bullet\}$: disj. arcs with positive flow in dual solution of timing problem.

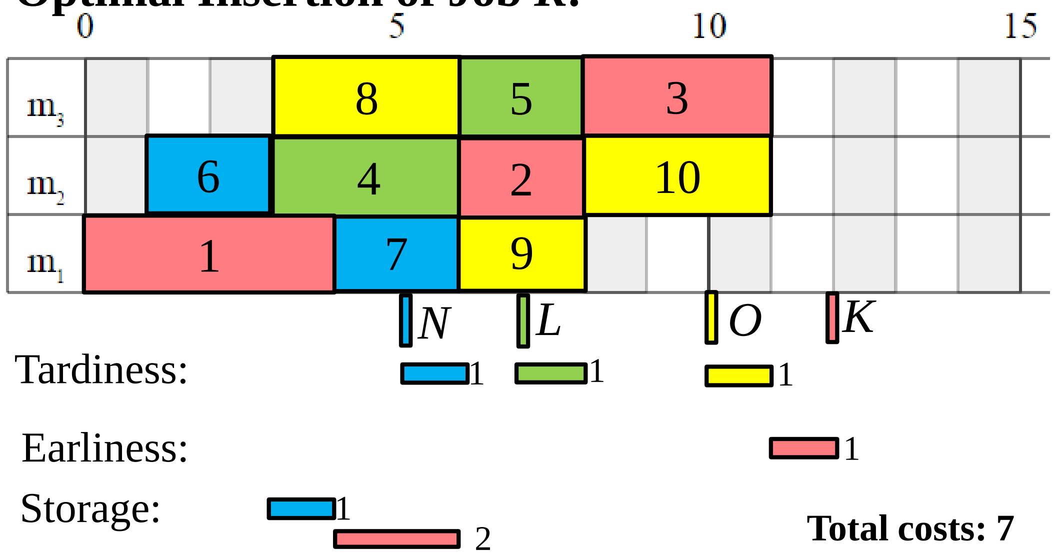
$$\sum_{v \in I^{\text{crit}}} x_v \leq |I^{\text{crit}}| - 1$$

In the example, we generate "just" 12 insertions.

Initial solution:



Optimal Insertion of Job *K*:



Theorem
Opt. Job Insertion Algorithm exactly solves the optimal job insertion problem.

$$\text{minimize} \quad \sum_{v\in E^J} \boxed{c_v x_v}$$

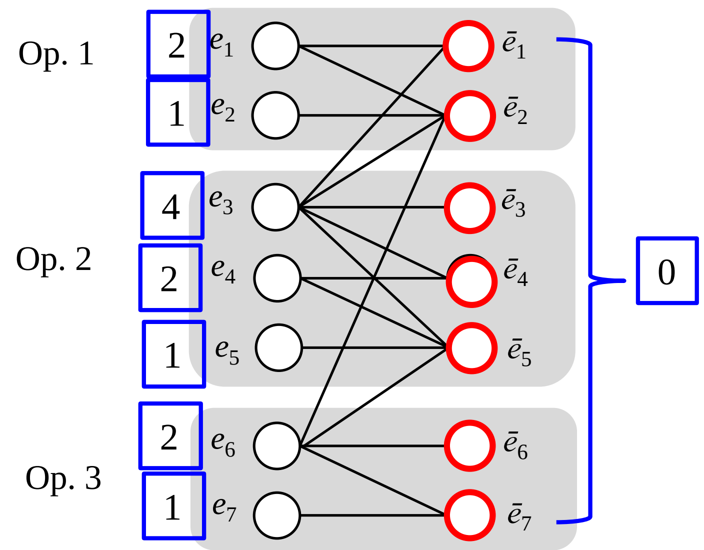
subject to:

$$x_w + x_v \le 1 \quad \text{for all } \{v,w\}\in U$$

$$x_v + x_w = 1 \quad \text{for all } \{v,w\}\in \mathcal{E}^J$$

$$x_v \in \{0,1\} \quad \text{for all } v\in E^J$$

- However, time consuming for medium and large problems
- Locally improving
  - Use weights to generate local neighbors, and stop after a certain time (or number of generated insertions)
  - Obtaining a locally improving neighborhood
- A neighborhood (basic idea): Generate a locally improved neighbor for a subset of jobs and go to best neighbor



4.3. Some Preliminary Computational Results

- Tabu search with swap-based neighborhood
  - A neighbor for each critical arc
- Tabu search with locally improving neighborhood
  - At most 4 jobs are extracted and reinserted
  - At most 150 insertions are considered for each job
- Clearly, tabu search iterations are time consuming

TS iter. / 100 sec.	20 jobs x 5 mach.	20 x 10
reg. objective	50000	3000
swap-based neigh.	140	30
locally improving neigh	4	1.5

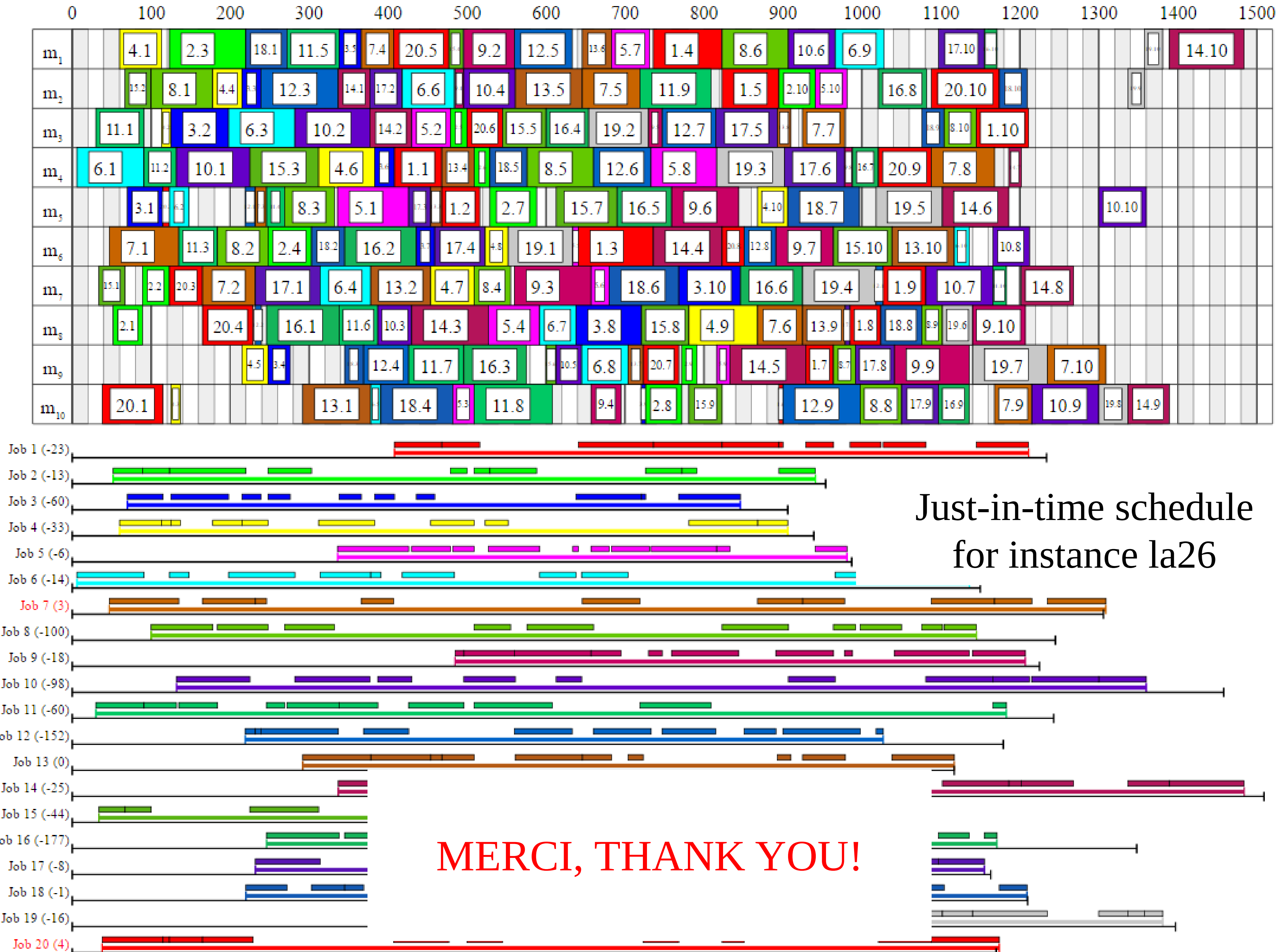
- Hence, very important to
  - Start tabu search with a good initial solution
  - Select moves wisely (and improve implementation!)
- In our example: just-in-time job shop scheduling with squared tardiness costs and linear storage costs
  - Main component are tardiness costs → use solution approach for job shop with regular objectives
  - See, e.g., Bürgy, R. (2016). *A neighborhood for complex job shop scheduling problems with regular objective*. Les Cahiers du GERAD No. G-2016-34. Montreal, Canada
  - Total comp. time 2400 sec. (600 sec. for initial solution computation)

- Comparison with straightforward MIQP
  - MIQP solves some of the smallest instances (la01-la03) to optimality
  - Poor solution quality for larger instances (la26-la40)
- Swap-based neighborhood has a quite good performance
  - (Near-) optimal results in smallest instances
  - Significant improvement of "initial solution" (up to 50%, depending on "tightness" of due dates)
- Locally-improving neighborhood
  - Quite good results in small instances, but "moves too slowly"
  - May be combined with the swap-based neighborhood (adaptive neighborhoods)

Concluding Remarks

- Importance and difficulty of scheduling increases
  - Automated production systems (robots!)
  - Versatile machines, e.g., additive manufacturing (3D printers)
  - Mass customization ("batches of size 1", upward shift of the order penetration point)
  - Smart Manufacturing: pushed by US government (https://www.manufacturing.gov/), Germany (Industry 4.0, http://www.plattform-i40.de/), and others
- We established general models and methods for job shop scheduling
  - First, to the best of our knowledge, considering convex tardiness, earliness, and storage costs
  - More complex process features can be considered as well
- Future work
  - Just-in-time job shop scheduling: Improve details (implementation, parameters, etc.) and use parallelization techniques
  - Apply methods to interesting problems in practice

Just-in-time schedule for instance la26



MERCI, THANK YOU!