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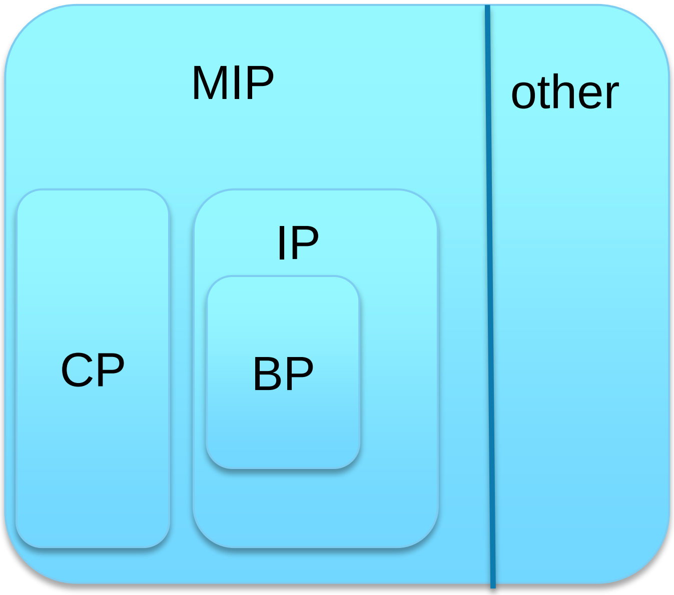
Optimization Problems

- General (non-linear) program (NLP)

$$\begin{aligned} \max\ & f(x) \\ \text{s.t.}\ & g_i(x) \le 0, \quad i = 1, \ldots, m \\ & x \in \mathbf{R}^n \text{ (and } x \in S) \end{aligned}$$

Optimization Problems



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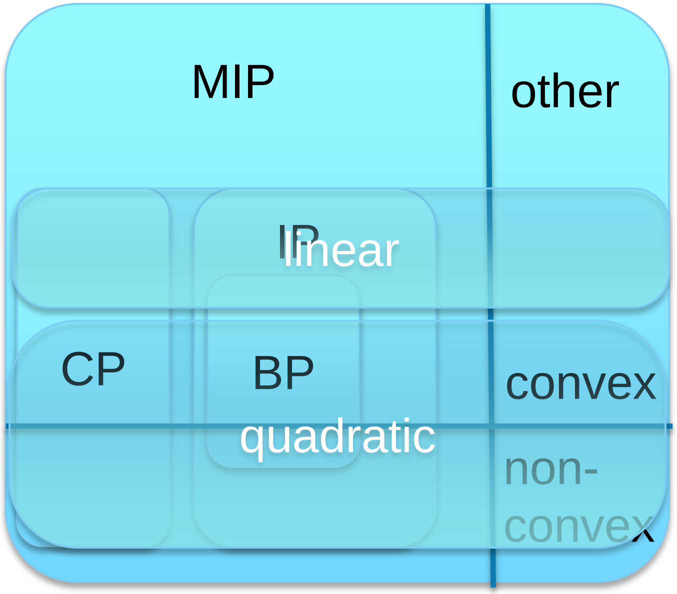
$$\begin{aligned} &\max \ f(x) \\ &\text{s.t.} \quad g_i(x) \leq 0, \quad i = 1, \ldots, m \\ &\qquad x \in \mathbf{R}^n \ (\text{and } x \in S) \end{aligned}$$

- What is S?
  – Mixed Integer program: $S = \mathbf{R}^m \times \mathbf{Z}^l$, i.e. some variables are integer
  – Integer program: $S = \mathbf{Z}^n$, i.e. all variables are integer
  – Binary program: $S = \{0,1\}^n$, i.e. all variables are binary
  – Continuous program: no S, i.e. all variable are continuous

Optimization Problems

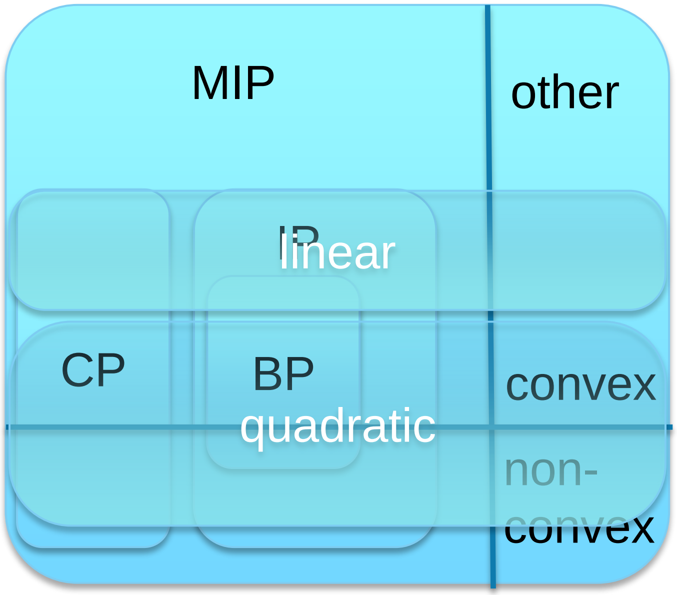


- General (non-linear) program (NLP)

$$\max\ f(x)$$
$$\text{s.t.}\quad g_i(x) \le 0, \quad i = 1, \ldots, m$$
$$x \in \mathbf{R}^n \text{ (and } x \in S\text{)}$$

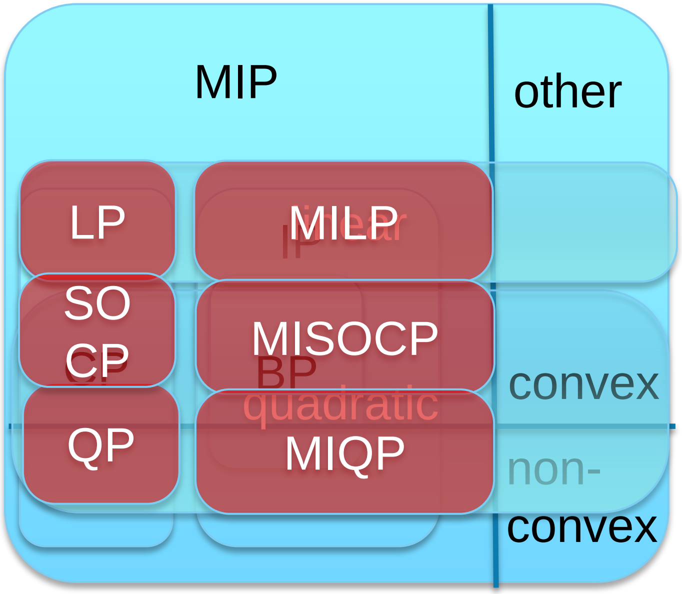
- How are f and g_i?
  - Non-convex
    - Quadratic: QP / QCP
  - Convex
    - Quadratic: QP / QCP
    - Linear: LP

Optimization Solvers



- Different algorithms for different special cases
- Algorithms for more general case often employ special case as subroutine
- Evolution of solvers from special problem to more and more general problems

CPLEX Optimizer



- LP: simplex, barrier (crossover), shifting,

  network, concurrent
- Convex QP: simplex, barrier (crossover)
- Local Non-convex QP: Barrier (no crossover)
- Convex QCP/SOCP: barrier (no crossover)
- MILP:
  - Branch-and-cut/Dynamic search
  - Benders (MIP only)
- Convex MIQP: Branch-and-cut/Dynamic search
- Global Non-convex QP and MIQP: spatial branch-and-bound.
- MIQCP: Outer approximation, nonlinear branch-and-bound

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